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Teaching and Learning Mathematics in Singapore

ASSOCIATION FOR SUPERVISION AND CURRICULUM DEVELOPMENT (SINGAPORE)

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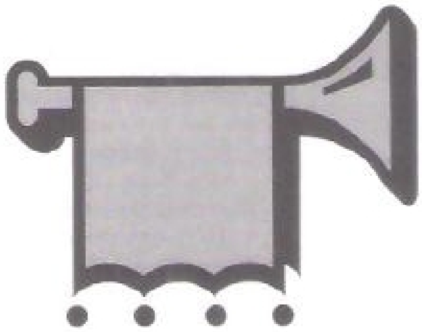

A Call for Articles...

The ASCD (Singapore) REVIEW Committee seeks original articles on teaching and learning...

Manuscripts should be between 2000-2500 words, typewritten (preferably Microsoft Word document) and submitted in the form of a hard copy together with a 3½ inch diskette. Photographs would be appreciated. Contributions may be addressed to:

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The themes for the forthcoming issues are:

Vol. 10 No. 3: **Project Work: Teaching, Guiding and Assessing**
Deadline for articles: 30th Nov 2001

Vol. 11 No. 1: **Helping the Underachievers**
Deadline for articles: 31st Dec 2001

Vol. 11 No. 2: **Beyond Class Time: Educating Critizens of Tomorrow**
Deadline for articles: 31st Mar 2002

Teaching and Learning Mathematics in Singapore

Vol. 10 No. 2

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*The choice of Mathematics as the theme for this issue of **ASCD REVIEW** is deliberate. ASCD Singapore sent a team of educators to Boston for the ASCD Annual Convention in March this year to make a presentation on the teaching of Mathematics in Singapore. Singapore's fine performance in the Third International Mathematics and Science Study (TIMSS) had many international educators asking Singaporean teachers they meet, "What's your secret?" As you can see from the range of articles shared in this issue on the subject, there is no secret - only sheer hard work and some creativity from the teachers to make the subject come alive!*

*Our next issue will focus on **project work:** the sharing of ideas on how best to guide our students through the learning process and in applying the skills we want them to master as they work their way through their projects. We would also like to look at ideas for planning for and assessing projects within different subject areas, age levels and differing curriculum needs. We encourage all educators, locally and abroad, to share with us whatever thoughts and ideas they may have on this topic. We will also be publishing on the theme of **helping the underachiever** in a subsequent issue. Hoping to hear from you soon.*

Soo Kim Bee
Editor

Singapore's Approach to the Teaching of Mathematics – a TIMSS Success Story

Diana Tang and Irene Ong

Abstract: Singapore was ranked first in mathematics and science in the Third International Mathematics and Science Study (TIMSS) and was equally successful in the TIMSS repeat study. Led by Mr Tan Yap Kwang, the immediate past president of ASCD, Singapore, Irene Ong and Diana Tang gave a presentation at the ASCD Annual Conference held in Boston, Massachusetts on 17th March 2001. This is an excerpt of the presentation.

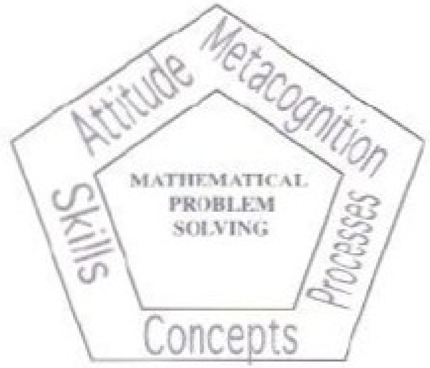
Introduction

The objective of the one and half hour interactive session was to share on our mathematics programme, teaching approaches, instructional resources and teacher development programme. Mr Tan covered briefly the education system in Singapore. Diana presented the key features of our mathematics programme and Irene shared from a practitioner point of view on how mathematics is taught in our schools.

Mathematics Education

Framework

The primary aim of our mathematics education is to develop student's ability in mathematical problem solving. This is done through five inter-related components as shown in the figure.



The components are development of conceptual understanding, mastery of essential mathematical skills, acquisition of mathematical and thinking processes, inculcation of positive attitudes towards mathematics and development of metacognitive insight.

Teaching Approach

Development of mathematics concept takes place spirally with topics sequenced appropriately across levels in increasing depth. This enables our students to maintain continuity in learning. For example in the teaching and development of 'Circles', pupils are first taught to recognise common shapes such as rectangle, square, circle and triangle. Pupils are expected to name the shapes by their mathematical terms when they see them. The concept of angles will then be built upon the concept of circles. The following activity is commonly used to help pupils get a sense of the size of acute angles. Firstly, pupils will cut semicircles from paper. Then the pupils will each fold a cut semicircle into half and then quarter to get the angle of 45 degrees. Alternatively, the teacher may get them to fold the semicircle into three equal parts giving an angle of 60 degrees. Using this folding pattern the pupils have created 'protractors'. This activity usually gives them a good sense of accomplishment.

Within the spiral approach, pupils progress from the concrete level to the pictorial level and then to the abstract level. This is evident in the coverage of topics from primary 1 to secondary 4. For 'Circles', concepts covered from primary 1 to primary 6 include shapes (with extension to semicircle & quarter circle), concept of angles (including 8-point compass), and circumference and area of circles. From secondary 1 to secondary 4, 'Circles' is treated in greater depth and breadth in 'Properties of Circles' and 'Radian Measures'.

Among the teaching strategies adopted by mathematics teachers in the classroom are the use of information technology (IT), group work or pair work and individual practice. The strategies adopted in the entire process of teaching and developing the topic are in line with the components in the mathematics framework.

Training & Professional Development

Training opportunities include pre and in-service training, local and overseas talks or courses as well as in-house sharing sessions and workshops. Professional upgrading is an on-going process and every teacher is entitled to 100 training hours a year.

A Teacher's Experience

On the average, our class size is about forty pupils to one teacher. In St. Hilda's Primary, pupils in primary 1 and primary 2 are of mixed abilities. From primary 3 to primary 6, high achievers are put into the same class to facilitate teaching. At the primary 6 level, we practise parallel teaching for pupils who have been identified to be very weak in Math. These pupils are grouped during Math Lessons to be taught by experienced teachers.

Classroom Activities

Our Mathematics instruction includes expository teaching, discussions and guided discovery. Many schools make use of teaching aids and manipulatives such as fraction discs, balances, weights, base ten sets, place value cards, clocks and others. Pupils are also exposed regularly to problem-solving sums, investigative work and enrichment activities in the form of games, puzzles and quizzes.

Mental Mathematics

Pupils are given verbal mental sums daily. Sometimes they may appear in the form of puzzles or brainteasers. Each class is allocated a Mental Sums period per week. Different schools have their own ways of rewarding their pupils for good performance in Mental Sums.

Abacus Programme

Teachers teaching in primary 2 and primary 3 are trained to teach the abacus. Learning to use the abacus enhances pupils' mastery of number bonds and concepts of place value. It makes the learning of addition and subtraction more concrete and improves pupils' concentration and mental calculation. Pupils learn five basic skills in addition and subtraction and one further skill of manipulating the beads for addition and subtraction involving multiple steps. Abacus quizzes and competitions are held in schools to enhance interest in the use of abacus.

Information Technology

Up to 30% of the curriculum time is set aside for the infusion of IT. In St Hilda's Primary, each class is allocated two periods a week i.e. an hour to have the lessons in the computer laboratory. Besides this, teachers also set up Learning Centres in the classrooms to provide more opportunities for pupils to explore and attempt different approaches to tasks and problems.

Enrichment Activities

We foster pupils' interest in a variety of ways. There are Mathematics Trails/Camps as well as National and International Mathematics Competitions.

Mathematics Club

Many schools have a Math Club as one of the school's co-curricular activities. The Math Club helps to promote pupils' interest in Math. Some activities carried out in the Math Club are solving challenging puzzles, playing math games, designing their own games, creating math problems, investigating number patterns and geometric shapes such as making 3-dimensional models.

In St. Hilda's Primary, pupils are encouraged to work on the enrichment activities in The Young Mathematician Card. Upon completion of the tasks specified in the card, pupils would receive a certificate and a Young Mathematician badge.

Mathematics Corner

In some schools like St. Hilda's, space under a staircase is converted into a Math Corner. Activities are held in this corner during recesses as well as before and after school. Parent volunteers also help out in the running of these corners. At the Math Corner they are known as Math Mums or Dads.

Conclusion

The presentation was followed by a Question and Answer Session chaired by Mr Tan. The participants showed great interest and asked many questions pertaining to our mathematics programme and instructional resources.

***Diana Tang** is a curriculum specialist in the Mathematics Unit of the Sciences Branch, Curriculum Planning and Development Division in the Ministry of Education.*
***Irene Ong** is a Head of Department (Mathematics) in St. Hilda's Primary School.*

ASCD Singapore in Boston

Singaporean educators making a presentation on the teaching of Mathematics in Singapore

Mr Tan Yap Kwang

Ms Diana Tang.

Mrs Irene Ong

Interacting with American educators.

Making sure the laptop is working.

An international audience listening to Singapore's presentation on the teaching of Mathematics.

ASCD Annual Convention 2001

Singaporean educators at ASCD Annual Conference, Boston, USA.

Le Roy Hay passes the gavel to Kay Awalt Musgrove, ASCD's 2001 – 02 President.

ASCD National Afiliate's Workshop facilitated by Bonnie Benesh.

At the ASCD President's Welcome Reception for foreign delegates.

'EXCEL' IN MATHS

V Soundara Pandian

Introduction

Teaching and learning Mathematics can be fun. Children want to learn and have fun when there is something different in the classroom. In this article, I propose to share how I used Microsoft Excel and newspaper clippings on Singapore economic slowdown to teach graphs to Primary 4 and 5 pupils.

Background

We have come a long way from the Survivor to the Efficiency driven education system to the Ability driven. I have gone through the various shades of our education system both as a student and as a teacher. As a teacher, at one time I was the *'sage on the stage'* but now, more than ever, I have come to believe that I must be *'a guide on the side'*, setting the stage for the students to learn. I believe in teaching not just for exams, although that is still a major component in the life of every student, but to build a strong character as well. "Intelligence plus character—that is the goal of true education" as Martin L. King Jr says. I subscribe to a model that enables me to build strong character in my students, "to equip them with the tools and the sense of opportunity to use their wits, skills, and passions to the fullest" not forgetting their individual differences. To me, learning is not a process where information is transferred to the student but rather a process where it is "analyzed, synthesized and personalized by the student". Like all effective teachers, I need a wide repertoire of strategies to achieve my goal. In this article, I intend to present and share a strategy in using the newspaper to teach Mathematics and a tool called Microsoft Excel. I believe Microsoft Excel is a vital tool in preparing the children for life in the new century. It enables the students to tabulate data and produce graphs at the hit of a button. Using newspapers in teaching is not a totally new strategy. Many teachers have used newspaper clippings as resources for project work and many such clippings have been used for information on classroom notice boards!

In the United States, NIE or Newspaper in Education program, first formalized in the 1930s, provides newspapers and teaching materials to classrooms. Today, there are more than 700 NIE programs internationally. In Malaysia, Star-Newspaper in Education has set up a web site to facilitate the NIE program. Research shows that NIE programs positively influence student motivation, attitudes, academic skills and fosters a better awareness of the world and of current issues. As a result, the student's achievement scores and reading skills show improvement.

But why use newspapers in teaching? The newspaper is an adult medium and students feel a sense of importance when using this resource. It bridges the classroom with the 'real world' outside. The newspaper cuttings motivate the pupils to read and provide a platform for discussion for the current national and social issues.

I felt that the news of the slowdown in the Singapore economy was important and would be of interest to the pupils. I was particularly keen to use the data (tables) as a resource for my pupils to tabulate, draw graphs, interpret, analyze and synthesize. The topic on Graphs in the Primary Mathematics Syllabus lends itself well to this strategy.

Simple skills in Microsoft Excel enable primary school pupils to plot graphs using any given data such as that on Singapore's economic slowdown. This would enable pupils to 'predict' future values and appreciate the implications of the slowdown. Although Primary school pupils are not required to plot graphs, this is easily done with the software. Teachers would only require a basic working knowledge in the software. They need not be Economics graduates either.

My Strategy

In Primary 4 Mathematics pupils use bar graphs and in Primary 5 they use line graphs. Pupils are required to interpret information from the given graphs; they are not required to plot the graphs. There is nothing wrong in using graphs which deal with school enrolment, sales of ice-cream cones, number of visitors in a park and the like. However, I wanted to relate their learning to real life and hence decided to use real data on Singapore's economic slowdown. This was something different from using the normal textbook in the classroom. In the process, pupils would also be able to appreciate current national issues. Furthermore, some of their families or relatives were affected by the downturn through retrenchment and loss of income. I am confident of the ability of my pupils to deal with major issues outside the classroom. But the burden was on me to set the stage and direct the focus of the pupils. For this, both the pupils and I needed to be very clear on the learning outcomes of the lesson.

(i) The Learning Outcomes

These are the learning outcomes of Primary 5 pupils.

Pupils will be able to

- present data in a table
- plot line graphs using a given set of data
- use simple skills in Microsoft Excel to do the above
- interpret the graph or obtain information from the graph
- interpret data given in a table (critical thinking)
- make predictions based on data given (creative thinking)

(ii) Preparing Data

This is the easy part of the lesson. I can obtain data on Singapore's economic status from the newspaper (The Straits Times) or from the web site of the Ministry of Trade and Industry.

SLOWING ECONOMIC GROWTH

GROSS DOMESTIC PRODUCT AT 1990 MARKET PRICES

% change over same period of previous year

	3Q '97	4Q '97	1Q '98	2Q '98
TOTAL	10.7	7.6	6.1	1.6
Manufacturing	9.8	7.8	6.1	-1.1
Construction	17.4	13.2	15.8	9.8
Commerce	9.2	3.2	1.4	-4.8
Transport and communications	10.9	8.2	6.4	5.4
Finance and business services	12.4	9.6	6.6	2.5
Consumer price index	2.3	2.3	1.1	[illegible]
Total demand	13.1	8.2	4.2	-5.2
Exports	8.7	9.1	9.8	-1.2
Unit business cost	-2.2	-1.0	-0.5	2.7
Net job creation	31,100	35,800	9,100	-2,000

FABIAN CHAN MIN-FAY

The figure on the right shows the newspaper clipping I used for a Primary 4 class. I directed the pupils to focus on the 'Total' figures in the first part of the lesson, and then, went on to build more understanding on the five sectors of growth. I did not use the data from the last 5 rows. Although I did not find a need to explain the meaning of 'Gross domestic product', I did take time to explain the way the year was divided into quarters: 3Q'97 for 3rd quarter 1997 (Jul-Sep), 4Q'97 for 4th quarter 1997 (Oct-Dec), 1Q'98 for 1st quarter 1998 (Jan-Mar) and 2Q'98 for 2nd quarter 1998 (Apr-Jun).

A similar chart (shown on the left) appeared in The Straits Times this year showing the overall figures for the year 2000, figures for 4th quarter 2000 and 1st quarter 2001. (Please note: '1Q 2000' in the graphic should be '1Q 2001'.)

SHARP SLOWDOWN

% change in growth rate from a year ago

	2000	4Q 2000	1Q 2000
◆ **Goods producing industries**	10.1	13.7	**1.9**
Manufacturing	15.2	18.8	**2.3**
Construction	-4.6	-1.9	**0.0**
◆ **Services producing industries**	6.9	8.7	**5.4**
Wholesale & retail	15.2	13.1	**6.9**
Hotels & restaurants	8.2	5.6	**1.5**
Transport & communications	9.0	6.8	**4.6**
Financial services	4.1	7.5	**3.0**
Business services	6.6	7.1	**5.4**
TOTAL GDP	9.9	11.0	**4.5**

Source: DEPARTMENT OF STATISTICS

NADYNE LIM

Before this chart appeared in the newspaper, I obtained data from the Ministry of Trade and Industry web site **(see Annex 1)**. From the data available, I produced my own table similar to the figure on the left **(see Annex 2)**. This is the chart that I used in a Primary 5 class to draw line graphs.

(iii) Using Microsoft Excel

A teacher needs to have only basic working knowledge of Microsoft Excel. He needs to know how to start the program, type in text and values in the cells, how to draw lines around the cells to produce a table, how to format the table and cells. He would need to know how to select cells from the table and plot graphs using the Chart Wizard assistance. He must know how to adjust the graph, save and print it. Additionally, he would need to know how to find the average of a set of numbers using the Average formula. This was to show pupils how calculations were made easy with this software. But the teacher must be careful not to confuse the pupils to think that the average reflects the economic growth for the next quarter. For teachers who are very new to this software, I have attached a simple step-by-step guide **(see Annex 3)**.

(iv) *Part 1 of Lesson*

In the first part of the lesson, I gave each pupil a copy of the data (table), and an example print out of a table and graph from Microsoft Excel. In the computer room, using an LCD projector, I took the pupils through the steps in Microsoft Excel to prepare a table, to plot a graph, and to print the table and graph (using the example print out as a model).

The pupils then worked on their own to prepare the table, plot the graph (bar graph for Primary 4 pupils and line graph for Primary 5 pupils), and print out the table and graph. They were required to use only the 'Total' figures (and not the figures for the sectoral growth). Additionally, Primary 4 pupils learn to round off the figures and Primary 5 pupils learn to average the figures.

From the graph, they were able to say if Singapore economy was doing well or badly.

(v) *Part 2 of Lesson*

In the next part of the lesson, pupils were encouraged to predict the total growth figure for the next quarter. Initially pupils made wild guesses. They were encouraged to bring newspaper clippings on the economic growth of Singapore. I have also made conscious effort to collect such clippings over a few weeks. Pupils then discuss in groups on the information from the newspaper clippings. They began to realize the impact of the economic slowdown and started talking about retrenchments, pay cuts, and effects on manufacturing, business and trade. The lesson then developed into a discussion on the sectors of growth (5 sectors in 1998, now classified into 7 sectors). With a better appreciation of the economic situation, the pupils attempted to make better growth predictions for the next quarter. For me, the actual figure was not important. I was more interested in the process that the pupils went through. The pupils drew another graph with their predicted value included.

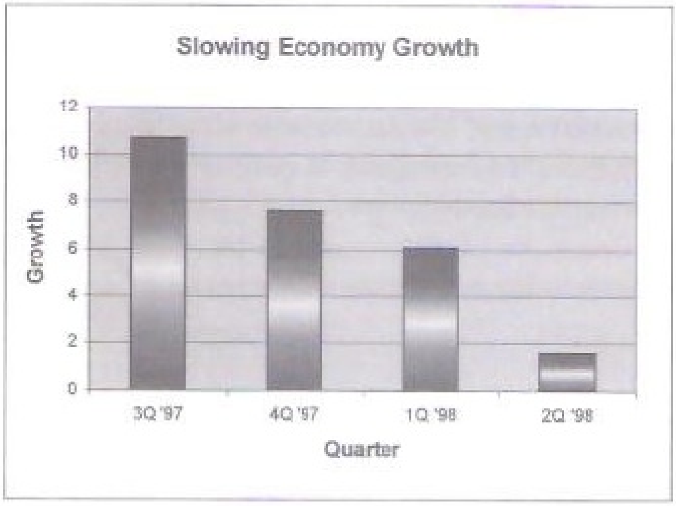
The pupils then went on to look closely at the figures for the various sectoral growth. They drew graphs for a sector they selected and interpreted what the graph meant.

(vi) *Extension of Lesson*

With information from the graphs and from the newspapers, pupils were encouraged to write their feelings about the economic slowdown in Singapore. Some wrote about the effects of economic slowdown in Singapore while some wrote about how they or their families were affected by the economic slowdown. Some even did a PowerPoint presentation to explain the slowdown to their classmates and some prepared posters to warn Singaporeans to prepare for the slowdown.

Example of a graph done by a Primary 4 pupil

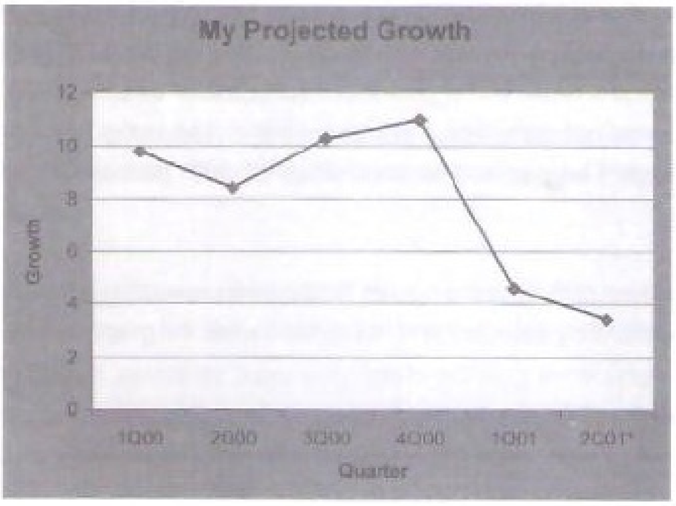
	3Q '97	4Q '97	1Q '98	2Q '98	3Q '98	Average
Actual	10.7	7.6	6.1	1.6		
Round Off	11	8	6	2		6.75
My Prediction	10.7	7.6	6.1	1.6	1.0	



Graph by
Amanda Lim
P. 4-2
Date: 14-8-98

Example of graph done by a Primary 5 pupil

1Q00	2Q00	3Q00	4Q00	1Q01	2Q01*
9.8	8.4	10.3	11	4.6	3.4



Graph by
Farhana Banu
P. 5D
Date: 18-5-01

Reflections

This has been a multi-dimensional approach – weaving in Mathematics, English Language, Social Studies, National Education and Art. Pupils picked up Microsoft Excel quickly as they were used to the Microsoft(r) Windows(r) environment. The pupils showed their enthusiasm and motivation by exploring further uses of the software such as using colours and changing fonts.

Pupils were taught new IT skills through their exposure to Microsoft Excel. They were given the opportunity to analyse real data and learn collaboratively with one another. The group work assigned to the pupils enabled them to recognize individual differences and have mutual respect for each other's point of view.

The pupils took pride in their ability to learn the use of new software and were proud of the work produced on the computer. As the pupils are often more familiar with the computer than I, they saw me as a co-learner and learnt the importance of lifelong learning. This was also brought home to them through the effects of loss of jobs and the necessity to learn new skills. Both the pupils and I enjoyed the lesson. It was heartwarming to see how the pupils responded to think of strategies to sail through the difficult period in Singapore's economic slowdown.

The major problem was in handling the large amount of newspaper clippings. It took some time and effort to sort and sift out the clippings and keep only very relevant ones. The whole lesson focused on income for Singapore from the 7 sectors of growth. The concept of Singapore budget or spending was not covered. This could be one area that could be incorporated in future lessons

On the whole I achieved the intended outcomes of the lesson which was not only to teach the required mathematics skills but also to expose the pupils to one facet of Singapore – the economic lifeline, and have fun in the process.

***Mr. V. S. Pandian** is the Head of IT and Media Resources and teaches upper primary mathematics in Jurong West Primary School.*

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Using Newspapers and Radio in English Language Teaching

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Connections between Teaching and Learning

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ANNEX 1

Sectoral Growth Rates

Per cent

Sector	1999	2000	1Q99	2Q99	3Q99	4Q00	1Q00	2Q00	3Q00	4Q00	1Q01
Percentage Change Over Corresponding Period of Previous Year											
Total	5.9	9.9	1.3	6.9	7.4	7.7	9.8	8.4	10.3	11.0	4.5
Goods Producing Industries	7.1	10.1	3.1	6.9	8.4	9.6	6.5	8.9	11.0	13.7	1.9
Manufacturing	13.6	15.2	6.5	14.6	16.7	16.2	13.2	13.2	15.2	18.8	2.3
Construction	-8.8	-4.6	-4.0	-11.8	-11.9	-7.6	-10.9	-3.9	-1.1	-1.9	0.0
Services Producing Industries	4.5	8.9	0.1	6.1	6.1	5.9	10.4	7.2	9.2	8.7	5.4
Wholesale & Retail	7.1	15.2	-2.3	6.6	9.4	14.7	17.3	15.3	15.4	13.1	6.9
Hotels & Restaurants	4.0	8.2	-2.2	4.7	5.5	8.0	10.7	6.8	9.9	5.6	1.5
Transport & Communications	7.1	9.0	5.8	6.3	7.8	8.2	9.9	9.7	9.5	6.8	4.6
Financial Services	0.8	4.1	-7.6	12.7	5.2	-6.5	11.6	-4.0	3.1	7.5	3.0
Business Services	1.5	6.6	0.3	0.9	2.2	2.5	5.2	6.5	7.4	7.1	5.4
Annualised Growth Rate – Seasonally-adjusted											
Total	5.9	9.9	4.4	16.9	1.9	8.0	13.0	10.8	9.8	10.3	-11.3
Goods Producing Industries	7.1	10.1	25.9	5.3	3.2	6.3	12.2	13.6	11.9	17.6	-27.9
Manufacturing	13.6	15.2	29.6	20.0	7.9	9.6	15.4	19.8	16.0	24.9	-37.1
Construction	-8.8	-4.6	21.9	-29.4	-11.1	-4.0	4.4	-4.4	-0.7	-5.9	11.9
Services Producing Industries	4.5	8.9	-5.6	21.4	0.4	8.4	12.6	7.9	8.4	5.7	-0.1
Wholesale & Retail	7.1	15.2	10.8	22.1	10.1	16.8	20.9	13.1	11.1	8.2	-3.8
Hotels & Restaurants	4.0	8.2	4.0	21.9	-7.4	15.8	15.2	4.9	4.5	-1.6	-1.5
Transport & Communications	7.1	9.0	5.8	8.2	8.2	11.0	12.2	7.4	7.8	0.0	3.5
Financial Services	0.8	4.1	-49.5	94.7	-22.0	-2.0	5.0	5.9	3.7	15.0	-10.8
Business Services	1.5	6.6	1.1	2.1	3.1	3.9	11.9	7.3	6.7	3.0	4.6

Source: Singapore Department of Statistics

SINGAPORE ECONOMIC GROWTH

	1Q00	2Q00	3Q00	4Q00	1Q01	2Q01
TOTAL	**9.8**	**8.4**	**10.3**	**11.0**	**4.6?**	
Manufacturing	13.2	13.2	15.2	18.8		
Construction	-10.9	-3.9	-1.1	-1.9		
Wholesale & Retail	17.3	15.3	15.4	13.1		
Hotels & Restaurants	10.7	6.8	9.9	5.6		
Transport & Communications	9.9	9.7	9.5	6.8		
Financial Services	11.6	-4.0	3.1	7.5		
Business Services	5.2	6.5	7.4	7.1		

Source: Ministry of Trade and Industry
10 April 2001

ANNEX 3

'EXCEL IN MATHS' GUIDE

Steps		Comments
1	Start Microsoft® Excel.	
2	Type text on 1st row.	GROWTH OF SINGAPORE ECONOMY
3	Format the text.	• Select the text. • Font: Arial, 14 pt, Bold in menu
4	Type labels on 3rd row.	1Q00 2Q00 3Q00 4Q00 1Q01 Average
5	Type values on 4th row.	9.8 8.4 10.3 11.0 4.6
6	Format value cells.	• Select cells, including cell below 'Average' • Click (in menu) Format —> Cells —> Number tab • Category: Number • Decimal Places: 1
7	Draw table lines.	• Select label row and value row. • Click Borders in menu
8	Format the table.	• Select label row and value row. • Font: Arial, 12 pt
9	Format label.	• Select label row. • Font: Arial, 12 pt Bold
10	Find average.	• Select cell below 'Average' • Type: =Average(• Click 1st value • Type: : • Click last value • Type:) • Press Enter

Steps	Comments
11 Draw graph.	• Select label row and value row (excluding Average). • Click Chart Wizard in menu • Standard Types tab • Select Line • Choose Chart sub-type: • (Line with markers displayed at each data value) • Next • Next • Chart title: Growth of Singapore Economy • Category (X) axis: Quarter • Value (Y) axis: Growth • Next • ⊙ As object in Sheet 1 • Finish
12 Adjust position and size of graph.	
13 Type source of data.	• (Ask teacher)
14 Type your name, class & date.	• Below graph
15 Save the file in your network folder.	• File name: GrowthXX • (XX = your register number) • [2nd file: ProjectedGrowthXX]
16 Print the file.	Select area to print Select File : Print preview Adjust margins, if necessary Print

Is There Only One Solution?

Wong Yim Kuan

"Is there only one solution?" the trainer posed the question to group after group of primary school pupils . The most common reaction the trainer received was ".... But we have found the answer." The trainer pressed on to get them to think of a second solution, to come up with as many solutions as possible, and to derive formulae.

The above was a typical scene which each of the maths five trainers encountered. I was one of the trainers.

The occasion was an enrichment class for solving non-routine problems for primary school pupils. We went to a total of 5 primary schools and conducted classes for the primary 4, 5 and 6 pupils. The schools included both mission and government schools. We had a total of 640 pupils of different learning abilities.

For each class, there could be as many as 10 groups of 4. To instill co-operative learning in the groups, roles were assigned to the four pupils – chairman, secretary, social control and logistics. The different duties for each role were explained and co-operation was emphasised. The *chairman* was to ensure that discussion was confined to the task at hand and to involve everybody. The *secretary* was to write down the solutions after the group had agreed and understood the solutions. The person in-charge of *social control* was to ensure that everybody in the group used a '10cm voice' to keep the noise volume down. The fourth person, in-charge of *logistics* was to collect the package of card and manipulatives and return everything in good condition.

The groups were expected to solve a puzzle together, finding as many solutions as possible. To assist them in their task, manipulatives were distributed for them to work out the puzzle. For the sake of those pupils who were not particularly good readers, the key points of the puzzle were explained to ensure they understood the problem.

An example of a non-routine problem given to the pupils is as follows:

You are asked to arrange the seating for a party. 3 teachers, 24 students, your principal and yourself are invited to the party. A triangular table seats 3 people, a square 4, a trapezium 5.

Using only squares and trapeziums, how many tables are needed?
Is there only one solution? Can you think of another ? Write down your answers and justify them.

Now, using only triangles and squares, how many tables are needed? How many solutions are there?

Multiple Solutions

For each of the classes, it was interesting to note that different groups came up with different solutions.

1. The manipulatives in the form of triangles, squares and trapeziums, came in handy for the pupils to experiment with the different combinations in a concrete manner. The concrete experience allowed them to count the number of people that could be seated and hence the number of trapeziums and squares needed. Through trial and error, the group eventually came up with at least one solution, if not more.

2. Another approach was to determine the number of trapeziums first. For example, if one trapezium is used, there will be 24 seats left to be filled. Since 24 is a multiple of 4, the number of squares required was easily worked out to be 6. Using the same approach, the group tried out using 2, 3, 4 and 5 trapeziums to see if it would yield a remainder that is a multiple of 4 to obtain yet another solution.

3. A third approach was to combine the seating capacity of one square and one trapezium, and the number of people seated was 9. The remainder was split into multiples of 4 and 5, which were the seating capacity of the square and trapezium respectively. This method quickly yielded two solutions.

4. For those pupils who were not sure if they had exhausted all the possible solutions, they were encouraged to draw a table and make inferences from it.

No. of people seated	Number used					
	1	2	3	4	5	6
Square	4	8	12	16	20	24
Trapezium	5	10	15	20	25	

Table 1

From Table 1 above, the answers were apparent. The two solutions of 4 squares and 5 trapeziums; and 6 squares and 1 trapezium were quickly deduced. It was also clear from the table that there were no other solutions possible.

Presentation

After 15 –20 minutes, the groups had completed solving the first puzzle and came up with at least one solution for each case. Most of the groups came up with two solutions. For each different solution, a volunteer came forward and explained the group's answer with the help of the manipulatives placed on the OHP or by writing on the whiteboard.

The mathematics puzzles were graded in terms of difficulty. Some were introductions to algebra leading the pupils to deduce the formula. Some were on permutations with real life examples. Some were on based on mathematics concepts of number patterns. At the end of the two hours, the groups solved a total of 4 puzzles including time to share their answers.

A little encouragement was needed to persuade the volunteers to come forward to share their solutions. The less articulate pupils were given help with their explanations.

Observations

The pupils were expecting step by step instructions to go about the problem. In the absence of a definite procedure, they were at a lost as to how to begin. It gradually became better with the second and third puzzles. The pupils were more concerned with procedural knowledge rather than product knowledge. The often repeated request was "how do we do it? " and expected to be told the 'how'. They were gently led to experiment with the manipulatives . They were guided to make deductions from what they saw.

Using manipulatives in a Maths class.

Using manipulatives to generate possible solutions.

As soon as one solution was obtained, the pupils tended to conclude that there was no more to be done. They stopped working, until they were challenged to look for another solution. It was the 'one-answer-correct' mentality at work. Getting them to think further and seek another solution was rather new to the pupils. Lacking in confidence as to whether the answer was correct, the pupils asked for the trainer's confirmation. Instead of giving a direct 'yes' or 'no' answer, the trainer challenged them to find out the answer themselves, albeit with help and prodding.

Although each puzzle, together with the questions, was printed on a card, the pupils hardly referred to it. They did not read the card until they were reminded to do so. The pupils had to be reminded to check against the questions to confirm if they had completed the task.

Writing down the solutions proved to be harder than we imagined. Again, in the absence of a definite way of writing down the answers, the pupils felt hesitant to record their answers. This was inspite of them being told to record the answers in whatever way that was clear. It took them a little while to know what to do and help was given when needed.

Group dynamics was interesting to observe. Since it was the first time they were assigned roles, many of them found difficulties in carrying out their roles in a definite way. However, they were aware of what each role should do and reported as such if, say, the child in charge of social control, made a lot of noise instead. A couple of groups went into conflict and intervention was necessary to restore harmony.

More importantly, it was observed that all the groups were totally on task. They were engrossed in solving the puzzle. The faster groups liked to be challenged and were eager to share their solutions with the class. The class was attentive to hear the different solutions to the same problem especially with the more difficult puzzles. They realised that different groups used different strategies : using the manipulatives, visualisation and abstract thinking using tables. Good thinking on their part prompted them to ask what if the tables were joined together? How would the results be affected? It prompted some good discussion.

As for the trainers, it gave us insights into the children's learning. Different pupils learn differently; they process information differently and each pupil constructs meaning for himself. It is important that we honour each different solution and encourage divergent thinking.

It was extremely heartening to listen to the pupils explaining how they obtained the answers, justifying them and applying similar strategies to the other puzzles. One group was even able to come up with the formula as the solution for the algebraic puzzles.

Learning Points

A little survey was conducted at the end of the session to as gather feedback from the pupils. The majority enjoyed the session. They learned to be co-operative and learned new ways of doing mathematics.

A sample of some of the comments from the pupils are:

- The puzzles allow us to use our brains
- Since the questions are tricky, we have to think creatively to get the answer
- It is fun and a very good learning activity
- Everybody has good solutions

Pupil explaining his method to the class.

To the trainer, such an approach opens an avenue for the pupils to think divergently. It encourages them to experiment, to seek more solutions than what is apparent. They learn to justify and make inferences. They know they could be different in their approach to solving the same problem. They master the confidence to explain their solution to the class. For social skills, they learn how to operate in a team and co-operate to attain the set goal.

To see the pupils' faces light up when they discover the solution to the puzzle is most satisfying. To see them helping each other to work out the solution tells us we have helped them to develop socially. To see the children excited about learning mathematics is the ultimate reward for all five of us.

***Wong Yim Kuan** was formerly a Secondary School Principal. She is now a trainer with MathLodge.*

e-Learning of Mathematics in Bukit Panjang Primary

Ng Teng Joo

Introduction

Technological advancement is progressing at such a rapid pace that it affects the way we live, work and ..... learn. Prime Minister Goh Chok Tong, during the launch of the Woodlands Regional Library, said that he would like Singaporeans to be unafraid of the new technology, and to reap the benefits brought about by advances in computers and the internet.

"But for that to happen, Singapore must first make building an 'e-inclusive society' a national goal."

Mr **Goh Chok Tong** (The Straits Times 24 June 2001)

Universities and major corporations all over the world are tapping the potential of IT by providing an e-learning environment to break the mould of the traditional method of teaching and training. Corporate giants like General Motors, PricewaterhouseCoopers, Dow Corning, Dell, IBM and Barclays are offering online management-education courses to their staff. In Singapore, the Nanyang Technological University has recently announced that it has embraced the promotion of the e-lifestyle in the campus.

" NTU aims to be a world-class university where learning and teaching will take on new and exciting forms with technological advances. Harnessing the potential of IT is a way of empowering our students. With the completion of a campus-wide wireless LAN, we have taken a giant step towards realizing our vision of Virtual Classroom, and we are able to provide a learning environment that breaks barriers of class and geography."

Dr **Cham Tao Soon** (NTU Link Feb 2001 Issue)

Schools are always looking for ways to formulate strategies to enable pupils to be continually effective in the future. The challenge therefore, is to produce students who are technologically savvy and are willing to constantly learn and improve themselves.

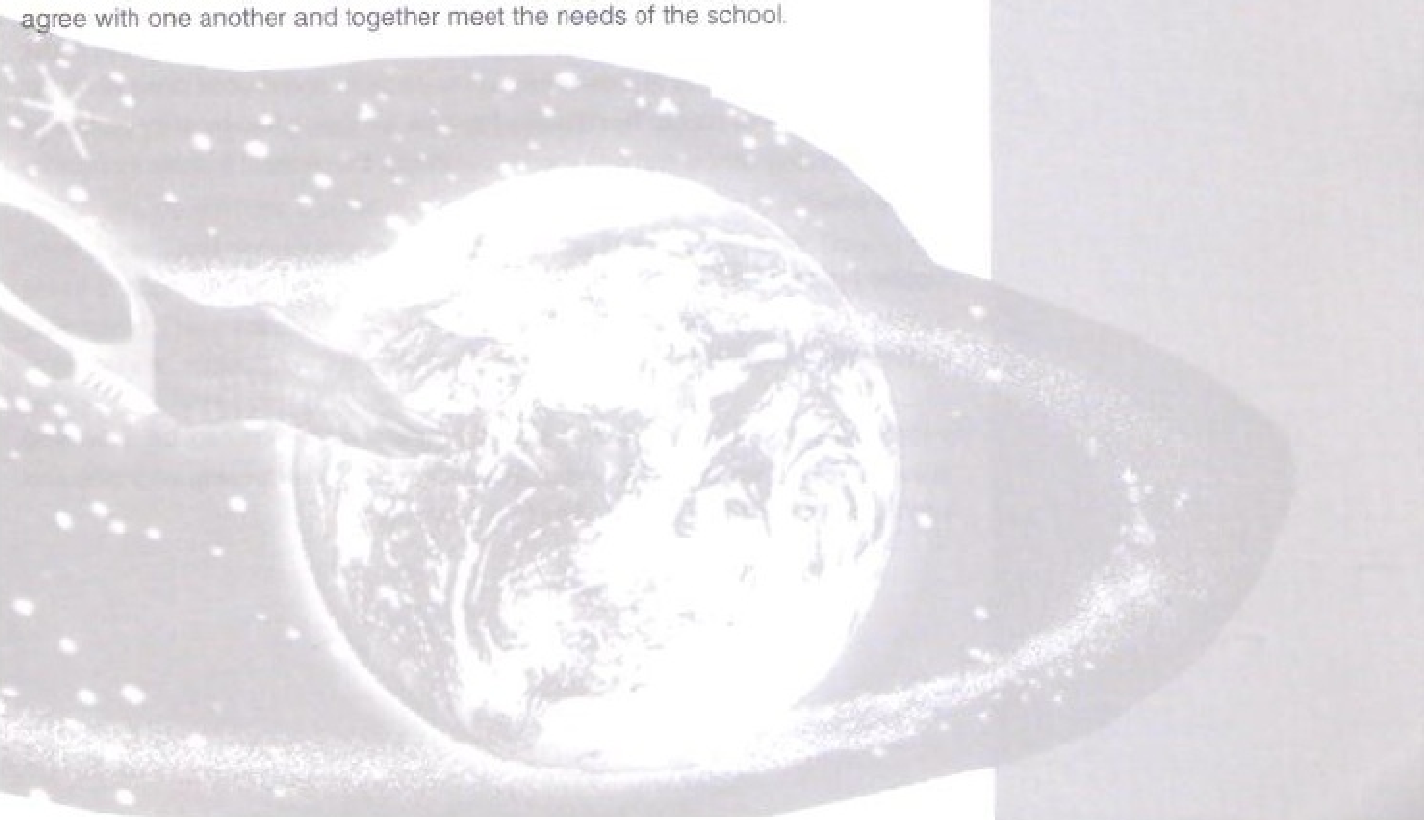
'e' for endeavour – Our School Motto

Online learning is incorporated into the curriculum in order for us to prepare our pupils for the knowledge-based economy. This is also in line with our mission to nurture pupils who are responsible for their learning. e-learning in mathematics is an innovative project that has been implemented in the school since April 2001. The feedback from stakeholders since then has been exceptionally positive.

Background

How did online learning for mathematics happen in Bukit Panjang Primary School? It all began when the school explored ways of harnessing the power of IT to serve the needs of our staff, about three years ago. We started by establishing a local intranet network whereby teachers could tap on teaching resources and communicate with one another and the administration. Teachers would exercise their professional judgment on how they could tap on IT resources for effective teaching. Once the structure was established and everyone saw the benefits of using the system, the next step inevitably was to consider how pupils could gain by learning through IT.

The experience to source for vendors and select one that would customize their packages to meet the needs of the school has been an exciting learning journey. Established content providers would rather we take their tremendous source of contents wholesale or not at all. Service Providers were not too keen to provide a customized platform for a primary school. We also required the two companies to agree with one another and together meet the needs of the school.



We required a platform that would help primary 3 pupils learn mathematics in a fun way. Animation and immediate feedback to guide pupils in getting the right answers would be essential. The automark system provides instant feedback for the teachers to assess how much pupils have understood the lessons taught. Frills like e-mailing and organizing modules meant for older students have to be trimmed. We also preferred to have the platform and content hosted in the web server of the Service Provider.

After several months of exploration, the school finally decided on a Content Provider that allowed our teachers to browse through its contents and choose those whatever was relevant to our pupils. The Service Provider we chose had recently partnered with a secondary school and was keen to customize the platform suitable for primary school pupils.

Online Learning of Mathematics

a) Selection of content

e-learning content refers to web-based courseware which can be accessed anywhere in the world from a simple web browser. We look for rich, engaging and interactive content as this is the key to e-Learning.

Our teachers have the flexibility in the selection of the mathematics questions from the content available. This allowed for differentiated 'homework' and caters for pupils with different levels of ability. Challenging thinking questions were incorporated for pupils in the high-ability classes, whereas pupils who are weaker at the subject would work on questions that emphasize the reinforcement of the basic concepts.

The topics selected for the online learning are designed to be in tandem with what the teachers have been teaching in the classrooms. The questions were changed once a week.

Unlike CD ROMs where the content is limited and 'static', online learning is more powerful as the content can be tailored and changed regularly according to the needs of the pupils. Instead of using IT (in this case, CD ROMs) to teach, we are facilitating pupils to learn through IT in a 'dynamic' learning environment.

Online learning has helped to make homework take a different form. Pupils would be exposed to a new dimension in learning and understand what it means to say that geography is no longer a barrier to learning.

b) Materials

- Computer
- Internet Access
- Telephone Line
- Printer

The downloading time depends on the computer processor speed, modem speed and the ISP line.

More important than any of the above is the time spent by parents to be with their children in doing homework via the internet.

c) Pupils *(without computers or internet access)*

Those who do not have the necessary materials are scheduled to come to school once a week and access 'homework' online in the IT learning centre. They spend about 2 hours doing their work under the supervision of a teacher and parent volunteer/s. There are cases in which parents would still prefer to send their children to school and access the 'homework' online even though they have all the materials at home.

Benefits to Pupils

- Any time any place – Pupils can do their homework anywhere in the world. Online learning provides a dynamic 'tutor' to-pupil interaction. The feedback is immediate and the 'tutor' is available anytime as long as the pupil has the necessary materials to log on.

- Learner-Centredness – The online system provides differentiated homework for the different ability groups of pupils. This approach maximizes the learning ability of pupils and motivates them to improve in the learning of mathematics.

- Convenience – Pupils can work at home and at a time most convenient to them when their parents are around to help.

- Valuable Experience – Pupils learn to realize that technology has transformed the traditional manner of learning to one which enables people to learn at their own pace without having to go through the conventional teaching process whereby geographical proximity (and other factors) could be a hindrance.

Benefits to Teachers

- Let the toolmakers do the work – Teachers can spend more time thinking of how to improve on their teaching. They can tap on the rich resources in order to teach the mathematical concepts without having to worry about preparing the powerpoint presentations, animations or attending to technical matters.

- Less marking to do – The automark system allows teachers to have immediate feedback on how their pupils have performed. The information from the item analysis helps teachers to identify the common mistakes and concepts that are not well grasped by pupils. This makes any remediation more focused and meaningful.

- Professional image – Online learning is not a fad. It is happening and going to be part and parcel of professional development in the future. Teachers who tap on IT (whenever appropriate) enhance their professional image in the eyes of the stakeholders.

Limitations and Concerns

- Accessibility – Pupils who do not have the internet access at home miss the experience and the opportunity to understand the meaning of online learning.

- Affordability – As long as the cost remains high and beyond the financial ability of the parents, schools with higher proportion of pupils from the lower SES background will not be able to introduce online learning on a bigger scale than others.

- Supervision – Homework for primary school pupils, unlike for matured students, needs constant supervision. Pupils might not take on the work seriously. They could click away mindlessly and do not give much thought in thinking so as to get the right answers. Not all pupils have good and conducive home environment to make online learning a rich learning experience.

- MCQ type only – So far the mathematics questions are confined to multiple-choice questions or short answers questions only. Essay type questions or problem sums are still elusive to assessment modules as marking such answers requires a highly intelligent natural language processing capability. These questions still have to be tackled in the traditional way – on paper.

Comments and Feedback

The pupils were more than receptive and the following are excerpts of their comments.

"Very nice, because it's very fun and very interesting."

"Because the computer when you do wrongly, it will tell you what to do."

"It has many games, if you don't know how to do, they will teach you – faster to learn (sic)."

(Source – http://sg.cna.mediacorpnews.com/articles/2001/05/14/singaporenews59689.htm)

The subject teacher is also very enthusiastic about the project.

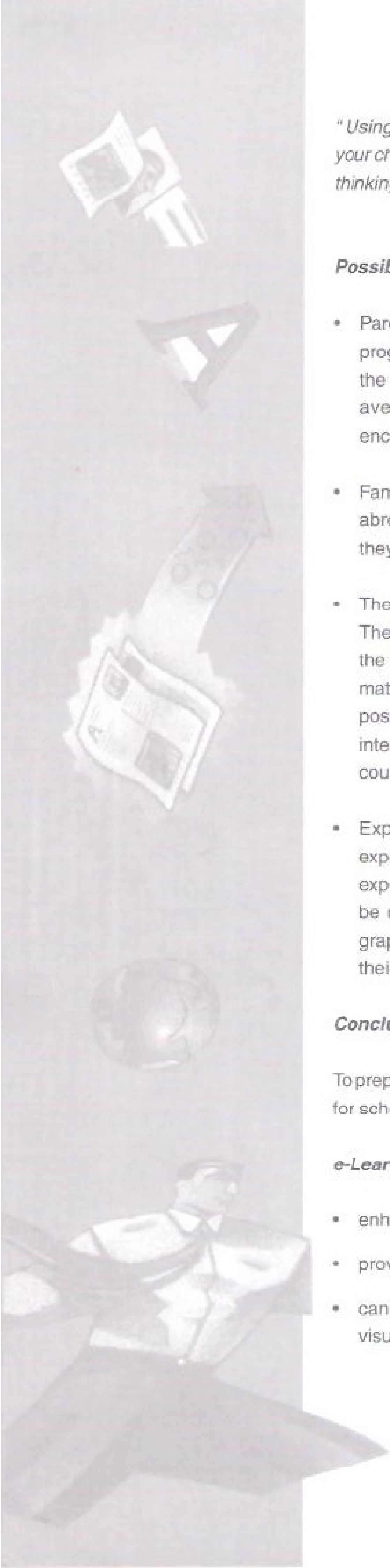
"So now, the computer is helping us to do the job of marking them, and actually guiding them in the questions they've done wrongly. That saves us a considerable amount of time."

Ms Belinda Ang – Maths Teacher

The following are the comments from the service providers.

"The internet has emerged to be a very cost effective means of communication and therefore presents tremendous opportunities for us to convert traditional training materials to web-based materials to ensure reach and access to continuous learning opportunities."

Inchone.com Pte. Ltd.



" Using carefully-structured educational online programmes, you can help to equip your child not only with academic knowledge but also life-long skills such as creative thinking and independent learning techniques."

i-tutor.net Pte. Ltd.

Possible Ideas – Possibility or a Dream?

- Parental Involvement – Parents are able to participate and monitor their children's progress. Accessed through a separate log-on and password, parents can review the performance of their children and compare the test results against the average scores of their schoolmates. This develops a supportive and encouraging culture at home.

- Families Working Overseas – Children who have to accompany their parents abroad need not miss school again! As long as they have the internet access, they can continue to 'attend' school wherever they are.

- The teaching of Science – Online learning is highly interactive and interesting. The rich resources and the flexibility in customizing a suitable platform allow the idea to be introduced to the teaching and learning of science. Just like mathematics, pupils can access science homework through the internet. A further possibility is for pupils to learn a topic in science all by themselves in a highly interactive lesson without the presence of the teacher. Perhaps, life-sciences could be learnt by pupils in this manner too.

- Experiments – Pupils will find learning meaningful and really exciting, as experiments need not be conducted in the laboratories. The virtual laboratory experiments can be conducted anywhere and anytime. The experiments can be repeated many times and the results instantaneously represented on the graphs. Pupils can draw conclusions from the experiments in the comfort of their own homes. It is cheap, convenient and safe.

Conclusion

To prepare our pupils to be part of an 'e-inclusive' society, introducing online learning for schools is timely and relevant.

e-Learning:

- enhances teaching methods which are mostly text-based
- provides the flexibility for the learner to work at his own time, pace and ability
- can be fun because it offers multimedia and interactive experience involving visual, audio and kinesthetic modalities

The vast amount of information available on the internet enhances the learning experience by keeping the pupil abreast of new information which cannot be found in textbooks or workbooks. These are real benefits rendered possible by harnessing the power of today's computers and plugging into the world of the internet.

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See Website below

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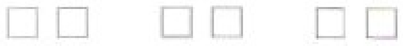
***Mr Ng Teng Joo** is the Principal of Bukit Panjang Primary School.*

Creating Misconceptions, Restricting Creativity?

Soon Yee Ping

"That is not right! Yours should be three groups of two..." Christopher was trying to tell Ariel that his configuration for '2 x 3' was wrong. "It should be like this, look ...one group of three and another group of three " echoed Nat. He used the tiles to show Ariel. The figure below shows the configurations exhibited by the children with coloured tiles to illustrate the meaning of '2 x 3'.

Christopher's and Nat's



Ariel's

This was what happened during one of the sessions I had with Christopher, Ariel and Nat. They are primary three pupils and they attend different schools. Christopher, Ariel and Nat are my friends' children. After end-of-year examinations, I was asked to conduct mathematics sessions with the children. The objective was to have fun and meaningful mathematics lessons. Here, I will be sharing with you three 'experiences' that took place during the sessions we had.

FIRST SESSION – Is Ariel wrong?

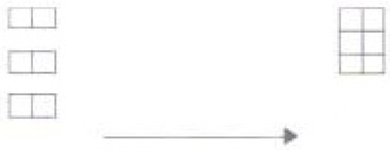
During the above session, the children were asked to show the meaning of '2 x 3'. With his two peers trying to show him "the" correct answer, Ariel had difficulty defending his solution. In addition, Christopher and Nat showed their mathematics text and exercise books to support their explanations. One of the rules of the session is the children have to justify their solutions. Another rule is they have to help me to critique each other's solution.

Multiplication has several meanings. The most common meaning attached to multiplication is "groups of equal quantity". Thus, the interpretation of ' 2x3' is 2 groups of 3. Christopher and Nat had clearly used this interpretation. Christopher and Nat claimed that Ariel was incorrect as this interpretation is the only one they knew. Was Ariel wrong then?

Let's look at another interpretation. '2 x 3' is usually read as 2 multiplied by 3. If this is so, then Ariel's configuration shows precisely the meaning of what is read. 2 repeated with a multiple 3.

Other meanings of multiplication

Multiplication has other meanings. Another way or model used to illustrate multiplication is the rectangle. The sides represent the multiplicands and the area represents the product as shown below:



2x3

This model is important. It will later help the children to understand the algorithms involved in the multiplication of two digit numbers, fractions, decimals and in algebraic expansion.

Another meaning given to multiplication is combination. Let's use an example to illustrate.

> *Mrs Tan is baking cookies for Christmas. She uses four shapes (circle, heart, star, and square) and three flavours (chocolate, oatmeal, almond). How many different types of cookies can Mrs. Tan bake?*

The question here is how many ways can you combine the four shapes with the three flavours. For each shape Mrs Tan can bake three different cookies of different flavours. Since there are four different shapes, we can have 4 x 3 types of cookies. The multiplication here has none of the meanings indicated above.

This last meaning of multiplication has important application in technology. It is found in combinatorics which is an essential mathematics topic covered in computer studies.

SECOND SESSION – *Squares are Rectangles?*

At another session, Christopher, Ariel and Nat were each given three coloured square tiles placed in a row as indicated. They were to address the question: "How many rectangles are there?" 

These were their solutions:

Nat's Christopher's & Ariel's

Christopher and Ariel said in unison "three". "I have one" commented Nat. He then had a peep of his two friend's solutions and nodded in agreement. He thought the square tiles had to stay intact and viewed the whole as one rectangle. Placing three square tiles as shown, I asked the children "What about these?"

"But these are squares, they are not rectangles" responded Christopher. When I said, "squares are rectangles", he said, " How can this be? They do not look alike." He went on to show me by pointing to the two longer sides and two shorter sides of the rectangle in comparison to the four equal sides of the square. "They are not rectangles!" Nat and Ariel supported in emphatic tones. They concluded again referring to authorities – quoting their teachers and their textbooks.

How did I solve this problem? I renamed the shapes as shown in Figure 1 below.

Square Oblong

Figure 1

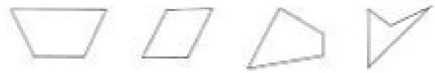


Figure 2

The following is the dialogue I had with the children.

What are your names?
Christopher, Ariel, and Nat

What is another name which I can use for each of you?
Boy.
Christopher is a boy, Ariel is a boy, Nat is a boy. – Christopher, Ariel and Nat are boys.
Square is a rectangle, Oblong is a rectangle – Squares and Oblongs are Rectangles.

What are common for both the square and the oblong?
Four sides, four corners.

What about these? *(I drew some quadrilaterals. These quadrilaterals drawn are not rectangles as indicated in Figure 2)*
They have four sides and four corners too.

What makes these shapes different from these? *(I pointed at the square, oblong and then the quadrilaterals)*
The corners. The corners are right angles.

Can you describe a rectangle for me now?
A rectangle has four sides and four right angles.

It was extremely difficult to convince the three boys. There is always a tendency for children to revert to their earlier conception that squares and rectangles are entirely different. I wonder whether it is advisable to present squares and rectangles as independent entities to children at an early age?

THIRD SESSION – What comes next?

I arranged coloured tiles in a line as shown and the question asked was "What comes next?"

■ ■ □ ■ ■

R B Y R B ....

Christopher and Nat both answered "yellow". Ariel said "Red". When I asked "why?" Nat said "Red, blue, yellow that is the pattern so after red and blue the next one should be yellow". Ariel saw red, blue, yellow, red, blue, as one whole pattern. To Ariel, the next tile should have been red since the whole pattern was going to repeat itself. Christopher interrupted, "Hey, I see another pattern, it should be red, then yellow, blue...see it is like a reflection!"

At this point, the children were excited. After some interesting explorations, Nat concluded, "So there are many answers as long as we can explain our pattern. Is this always so?" I presented the following example to the boys.

> *As Susan walked down a row of ten houses, she saw the following numbers on the houses*
>
> *3, 13, 23, 33, 43, 53, 63, ...*
>
> *What is the number of the next house?*

The three boys in unison answered 73. The question did not present an ambiquity to them. There was a context for the children to refer to.

Reflections & Questions

What have I learnt from the sessions with the three boys? I have benefited from these sessions. They caused me to reflect and ponder on my practice as a mathematics teacher.

1. Are we encouraging children to show us their perspectives on things?

Quite often as teachers, we have preconceived "answers" to the questions we put to children. In the "what comes next?" question, the expected answer is 'yellow'. Hence, there is a tendency for us to grade a child incorrect if the answer is not "yellow". By doing this, are we restricting the child to our frame of thinking and view of the world. If this is so, are we helping to encourage creativity, and to promote diversity in thinking? This applies to common classroom practice. We often give children a method of solving a problem and assign similar types of questions for them to apply that 'method'. How often do we encourage them to come up with their own solutions? Have we tried to enter the child's world and see with their eyes? As teachers, have we given ourselves the opportunity to look at a student's solution that we have not thought of or seen ? How often do you sit back to marvel at the creativity of these young minds?

2. *Are we presenting enough scope to show the correct meanings of the mathematics concepts?*

Intuitively, Ariel was correct on the meaning of multiplication with the configuration he presented. Christopher and Nat relied on what was taught as the only meaning, and concluded that Ariel was wrong. It was in that context, Ariel was graded wrong. As teachers, how often have we incorrectly graded a child's correct answer? Are we aware of the different meanings and interpretations of the concepts we presented to the children? By the limited scope of our presentation to the children, are we creating misconceptions? As in the case of the squares and rectangles, by the limited scope of our presentation to the children, we are creating misconceptions. In the assignment of exercises do we further reinforce this misconception?

3. *Are we helping the children to be the owner and authority of their learning?*

The three boys constantly refer to their texts, teachers and mathematics exercise books when they cannot explain their solutions. They often regard the teacher and text as the authority of the knowledge. In our teaching, are we encouraging the children to "mimic" us? Are they able to articulate their understanding – defending and convincing others of their knowledge? If they are just regurgitating what their teacher has told them, does this show that their knowledge still belongs to the authority – us the teachers?

4. *Have we provided a sound link for future learning of concepts?*

Mathematics is a network of concepts. Latter concepts are built on earlier foundations. In the example of multiplication, the area model provides a link for explanation of fractions, decimals and algebraic expansion at a later stage. Squares are not rectangles – a misconception which teachers have to rectify at a later stage to facilitate learning of other new geometrical concepts.

The three boys showed me their depths of learning through their articulations. I marvel at their enthusiasm and potential in learning. In all the sessions, I have to keep pace with this enthusiasm, with the new ideas and perspectives they presented and with the excitement they exude. I have to remind myself constantly, to carefully keep this fire of enthusiasm burning in them.

Thank you boys for providing so many insights into learning.

***Dr Soon Yee Ping** is a mathematics teaching consultant in private practice.*

BACK TO BASICS in Macpherson Primary School

Wang Chee Ting

Problem

Tests administered at the beginning of the year 2000 indicated that 41.6% of the 2000 PSLE cohort were not ready for the P6 Mathematics content. Diagnostic tests revealed that pupils were weak in number bonds; disinterested in the subject; had limited vocabulary and low language skills which compounded the pupils' difficulties. Specifically, pupils' difficulties in basic Mathematical skills pertained to:

[a] the concepts of number, numeration and place value notation
[b] the concepts of addition, subtraction, multiplication and division
[c] the processes and the strategies involved in the four operations
[d] the computational skills in the four operations

Teacher feedback reflected the following:
[a] pupils' fear of Mathematics
[b] constant failure and feelings of incompetence, discourage and demoralize pupils who are poor in Mathematics
[c] demoralized pupils feel it does not make sense to try

Objective

In redesigning the differentiated curriculum, teachers set incremental targets. To encourage active participation, the initial goal was to reduce the high percentage of U-grades. U-grades for the EM3 stream mean scores of <30 while that for the EM2 stream relate to scores <20. The table below is one practical illustration:

SET TARGETS	
P6 EM3	To reduce the percentage of U-graders from 100% to 50%
P6/6 EM2	To reduce the percentage of U-graders from 50% to 10%
P6/5 EM2	To reduce the percentage of U-graders from 93.5% to 50%

Solution

Teachers, empowered to make instructional decisions:

- banded the children according to Mathematical ability
- put aside conventional syllabi for one that addresses numeracy deficits

- adapted the curriculum to help pupils acquire the intended skills
- tailored a curriculum to ensure it does not contend with conflicting priorities
- delivered pedagogy that is embedded into the fabric of everyday practice
- played the role of creator, i.e. daily created a safe and trusting environment in which pupils can develop an awareness of what their role is to be
- made explicit to pupils that they are responsible for setting their own goals and criteria

Implementation

1. Mastery learning of basic content; teachers involved in the project, modified the assessments for pupils involved i.e.

Class	Assessment Level: P6EM2 (at CA1, SA1)	(at CA2, SA2)
6-5	P5 EM2 Content	P6 EM2 Core Content
6-6	P5 EM3 Content	P6 EM3 Core Content P6 EM2 Selected Topics

Class	Assessment Level: P6EM3 (at CA1, SA1)	(at CA2, SA2)
6-8	P5 EM3 Content	P6 EM3 Content
6-9	P3 Content	P4 EM3 Content

2. Individual attention was given to pupils with serious deficiencies. This strategy helped the teacher detect the obstacle that was affecting the child's progress. It was also effective in timely correction of a mistake made.
3. Small group instruction was given to pupils with similar deficiencies.
4. Fortnightly evaluation helped to steer the plan towards continuous improvement in pupils' Mathematics.
5. The evaluation focused on these domains:
   - Pedagogy
   - Resources for teaching and learning
   - Assessment

   5.1 **Pedagogy**: Classroom pedagogy was pupil-oriented. It hinged on teacher ability to employ a variety of skills to engage pupil participation.

   5.2 **Resources**: The administration lent its support with purchases to ensure appropriate curricular resources, which promoted the teaching and learning of Mathematics.

   5.3 **Assessment**: Practicals and formal activity-based lessons complemented the pen and paper exercises which were tailored to redress deficiencies.

Tangible Benefits:

1. Purposeful learning brought on changes in pupils' learning outcomes. (Refer to Annex 1)
2. Pupils end up using the Mathematics they are comfortable with rather than that which a teacher would expect.
3. Active learning encouraged better classroom management.

Intangible Benefits:

1. Pupils developed control over negative reactions to Mathematics.
2. Increased pupil self-esteem and confidence through success in Mathematics.
3. Tailored curriculum created enjoyment of Mathematics.
4. Pupil interest and active participation facilitated mindset change by both learner and teacher.
5. Teachers were convinced every child can learn.
6. The Pupil Welfare Co-ordinator in his thesis validated teacher observations of the intangibles – increased pupil self-esteem and confidence through success in Mathematics.
7. The success of this tailored curriculum served as a springboard for the school to design other tailored curriculums to cater to the different learning needs of the students in other subject areas.
   7.1 Teachers of P4, P5, P6, inspired by the positive impact, designed their version of Mathematics Back to Basics.
   7.2 Following the review of pupils' performance in March 2001, Chinese language teachers designed a Chinese Language Back to Basics for twenty Primary One pupils who scored less than 50 marks for the Chinese Language in March 2001. The outcome at the end of 10 weeks, motivated both teacher and pupils. (Refer to Annex 2)

Inspired by the positive impact, this year, 2001, the principal and her staff at **Campus Two (Woodsville Primary School)** re-designed the Mathematics curriculum to very satisfying outcomes. The data in Annexes 3 and 4 reflect the positive outcomes.

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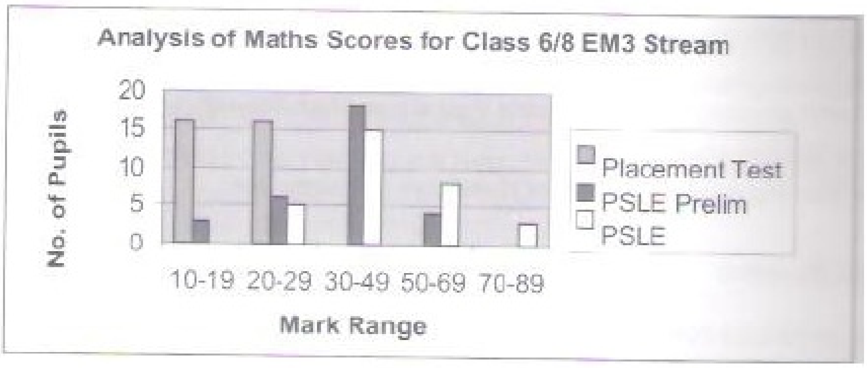
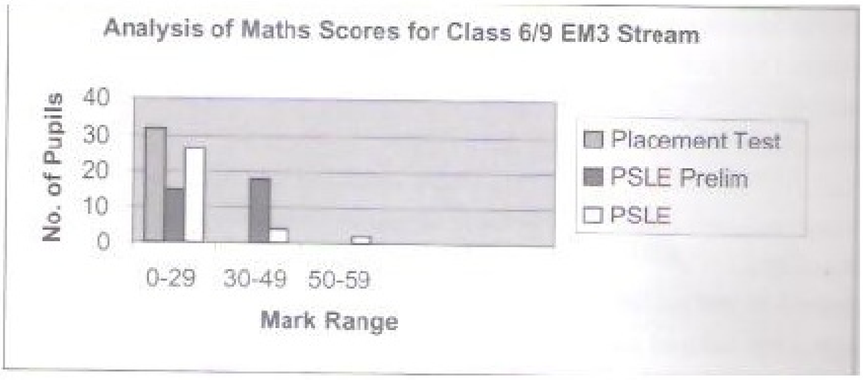
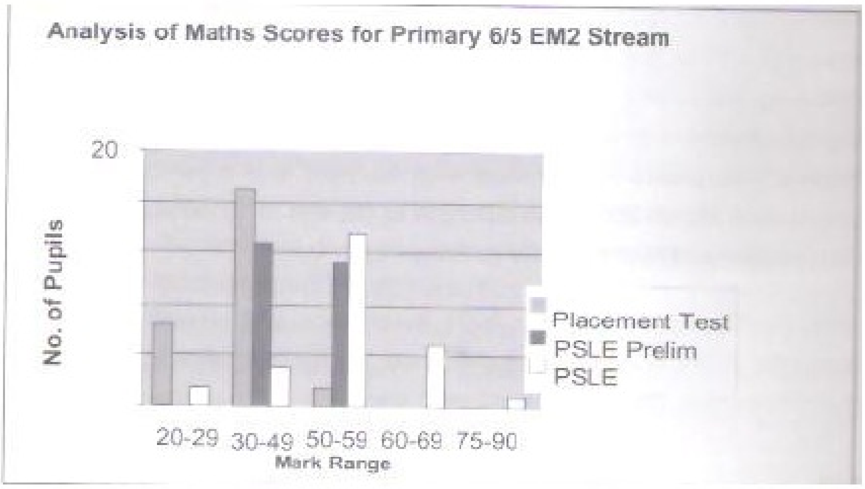
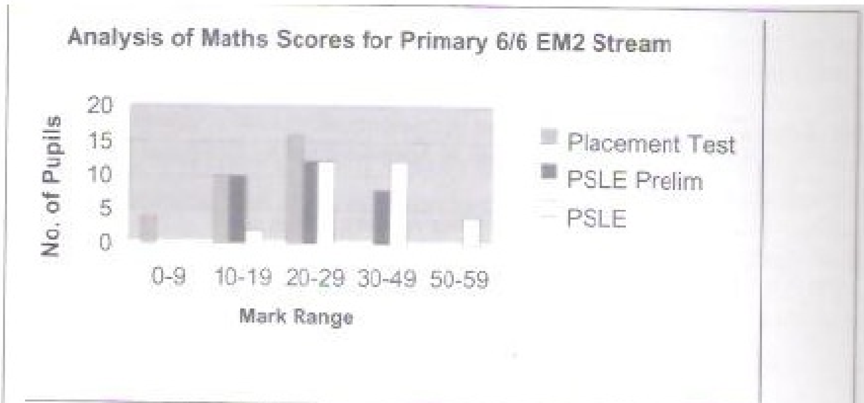
ANNEX 1 (Maths Back to Basics in MacPherson Primary School









U-graders reduced from 23% to 6.7%

ANNEX 2

Assessment Tools for Chinese Language Basics

Assessment Mode	*Assessment Tools*
Semestral Assessment 1	• Combination of P1 SA1 and P2 CA1 papers • **P3 CA1 paper**
Continual Assessment 2	• Combination of P2 CA2 and P3 CA1 papers • **P3 SA1 paper**
Semestral Assessment 2	• P3 CA2 • **P3 SA2 paper**

PROGRAMME OUTCOMES

MacPherson Primary School: Chinese Language Back to Basics						
No.	**Name**	**CLASS**	**CA1**	**SA1**	**CA2**	**SA2**
1	SUCIPTO LIM YONG SENG	1/1	30	54		
2	EPSON NGANGI	1/2	42	62		
3	HO XUE QI SHIRLEY	1/3	60	46		
4	LIM LI TING	1/3	44	32		
5	MAUNG KAUNG HTET TUN	1/3	44	48		
6	NG SI HUI	1/3	56	31		
7	TAN YUN ZHAO	1/4	50	43		
8	WANG JIA SIANG	1/3	30	30		
9	YONG THANAKAN	1/3	66	29		
10	TAN SU SIN	1/4	32	54		
11	CASSIDY YAP HAN YANG	1/5	50	15		
12	AW KEE LI	1/5	42	39		
13	HO GUO WEI	1/5	68	38		
14	LEE SHI KAI	1/5	72	50		
15	PHANG WEE LONG	1/5	38	43		

ANNEX 3 (Maths Back to Basics in Woodsville Primary School)

Assessment Level

Stream	Class	Assessment Level	
		At CA1	At SA1
6EM2	6B	6EM3	6EM3
	6C	5EM3	5EM2

Stream	Class	Assessment Level	
		At CA1	At SA1
6EM3	6D	P3	5EM3

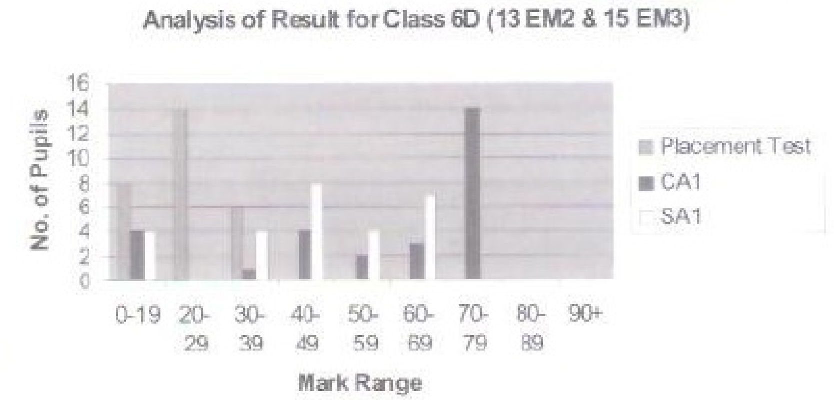
Targets

6B (6EM2) To achieve 100% at PSLE
6C (6EM2) To achieve 75% at PSLE
6D (6EM2) To achieve 55% at PSLE

The Re-designed P6 Maths Curriculum in Woodsville Primary School

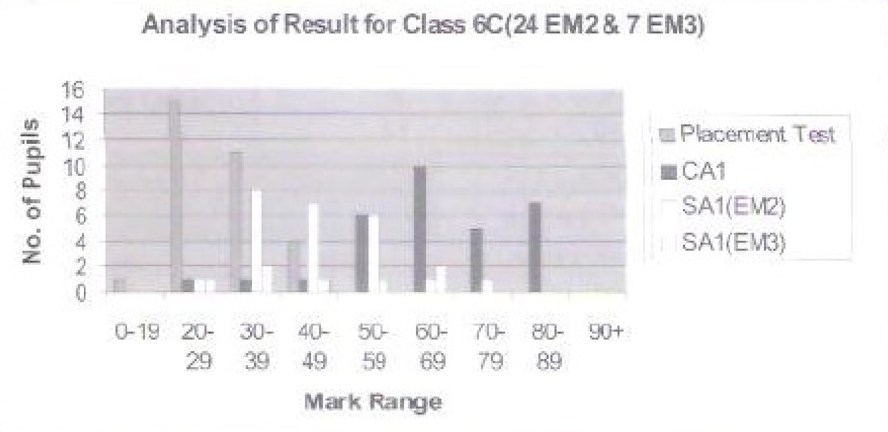
Class	Term1	Term2	Term 3
6B – 22 EM2 – 7 EM3	5 Weeks – P6 EM3 Book A	9 Weeks – P6 EM3 Book B	10 Weeks – P6 Core Topics • Fractions • Ratio • Percentage • Decimals • Whole Numbers • Measurement • Graphs/Pie Charts • Angles (basic)
6C – 24 EM2 – 7 EM3	5 Weeks – P5 EM3 Book A – P5 EM3 Book B	9 Weeks – P5 EM2	2 Weeks – P5 EM2 8 Weeks – P6 Core Topics • Fractions • Ratio • Percentage • Decimals • Whole Numbers • Measurement • Graphs/Pie Charts • Angles (basic)
6D – 13 EM2	5 Weeks – P3	2 Weeks – P3	3 Weeks – P5 EM3 Part 2
– 15 EM3		7 Weeks – P5 EM3	7 Weeks – P6 EM3 Core Topics

Class	Mark Range	Placement Test 1.2.2001	CA1	SA1
6D – EM2/3	0-19	8	4	4
Enrolment (28)	20-29	14	0	0
– 24 EM2	30-39	6	1	4
– 7 EM3	40-49		4	8
	50-59		2	4
	60-69		3	7
	70-79		14	0
	80-89			
	90-99			
	Total:	28	28	27*



U-graders decreased by 50%
Number of passes increased from 0% to 67.9% in CA1 & 40.7% in SA1

Class	Mark Range	Placement Test 1.2.2001	CA1	SA1 EM2	SA1 EM3
6C – EM2/3	0-19	1	0	0	0
Enrolment (31)	20-29	15	1	1	1
– 24 EM2	30-39	11	1	8	2
– 7 EM3	40-49	4	1	7	1
	50-59		6	6	1
	60-69		10	1	2
	70-79		5	1	
	80-89		7		
	90+				
	Total:	31	31	24	7

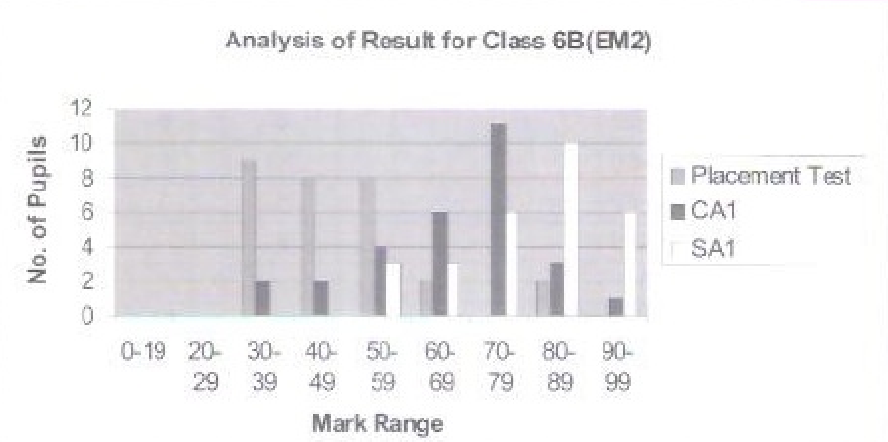


Number of passes increased from 0% to 90.3% in CA1
33.3% passed SA1(taking 5EM2) paper

ANNEX 4 (Maths Back to Basics in Woodsville Primary School)

Learning Outcomes

Class	Mark Range	Placement Test 1.2.2001	CA1	SA1
6B – EM2	0-19	0	0	0
Enrolment (29)	20-29	0	0	0
	30-39	9	2	0
	40-49	8	2	0
	50-59	8	4	3
	60-69	2	6	3
	70-79	0	11	6
	80-89	2	3	10
	90-99	0	1	6
	Total:	**29**	**29**	**28***



Number of passes increased from 41% to 100%

Teaching and Learning Mathematics: Suggestions for a Pre-school or Lower Primary Class

Seng Seok Hoon

Current views of mathematical learning and teaching focus on the child as a responsible student who attends to instruction and who constructs what is taught in a personal and meaningful way. The learner is an active agent in the teaching-learning process and the role of the teacher has moved from being one who imparts knowledge to one who helps students learn about learning. Contemporary mathematics educators not only want students to acquire considerable knowledge and skill, they also want students to think like mathematicians.

Most children have a natural ability to think mathematically. By age three or four, they start to use a number system and learn how to count which is an easy but important skill. They can also rank and arrange objects. According to Leeb-Lundburg (1985) logical thinking in children develops along with their mathematical discoveries e.g. at the age of two or three, they know that one block on top of another equals two blocks. Children learn with their senses and their whole bodies and the majority of these experiences happen at home and their neighborhoods. In their elementary school, children acquire a range of strategies solving addition and subtraction problems and even apply these strategies in other contexts.

Mathematical Knowledge and Skills

Brynes (1996) believed that children who are good at mathematics have a lot of mathematical knowledge and they can also engage in effective mathematical thinking. Mathematical knowledge can be split into conceptual and procedural knowledge. In the elementary school, some examples of conceptual knowledge include forming categories of mathematical entities (eg fractions, ratios, acute angles) and comprehending relative numerosity of groups of objects. On the other hand, procedural knowledge refers to the steps needed to attain specific mathematical outcomes (eg to count, add, subtract, divide, multiply or to find the area of a rectangle).

Mathematical ability consists of these two forms of knowledge and being good at mathematics does not bank on knowledge alone. The student must know how to use mathematical knowledge to solve problems. At different age groups, Brynes (1996) noted that different mathematical skills are being mastered and it takes a long time for a child to think like a mathematician. Even after many years of mathematics at school, some students have a very shaky grasp of simple arithmetic.

In his book "The Mathematical Brain" Butterworth (1999) emphasises unravelling the nature of mathematical skill and its relationship with brain function. He argues that mathematics teaching would fail unless facts and procedures are integrated with understanding and concludes that "understanding means being able to transform the core of the problem into collections and numerosities, and the various solution strategies implied by these representations."

Most of the studies that have examined what preschoolers know have focussed on either conservation of number or counting. According to Leeb-Lundberg, mathematics is more than counting. Real understanding of mathematics can only develop if children have plenty of opportunity to play in ways that will give them the foundation for later abstract thinking. Many children never develop this good sense of numbers because they are rushed into using the symbols and terms that signify an understanding of mathematics to many educators and parents. (Checkley, 1999). Many teachers in preschool have little knowledge of how to introduce the language of mathematical processes in the classroom. They must try to build and expand on the mathematical ideas that children are involved in through their daily activities.

Teaching and Learning Mathematics in Singapore

Mathematics is a compulsory component of the primary school curriculum in Singapore. Key mathematical topics and concepts are given in a series of teachers' guides and worksheets prepared by the Ministry of Education. Teachers are advised to select the appropriate topics and concepts in their instruction. There is a high element of freedom for this choice and it is usually done by a group of mathematics teachers under the guidance of the mathematics head of department within a school.

One of the challenges facing the mathematics teacher is to teach mathematics creatively. Tan (1998) singled out at least six components in creative mathematics teaching. The first component comprises ***basic pedagogical skills*** such as lesson planning, classroom management, communication and evaluation. The second component refers to the ***content mathematics***, creative techniques and knowledge of developmental processes. The third component is related to the competence in selecting appropriate ***assessment modes***. The fourth component refers to teachers' and pupils' ***motivation***. The fifth and sixth components are related to the ***learning climate and environment, educational policies and the school culture.***

In Singapore, educational policies influence the school learning climate. Since the early nineties, the mathematics curriculum has been officially developed and refined and has been used as a guide for teachers to plan their maths program. Teachers are not bound by the choice and sequence of topics presented but are encouraged to exercise flexibility and creativity and to ensure that the linkage within the curriculum are maintained. Recommended big changes have taken place at the pre and primary school levels and these include project work, multiple modes of assessment, creative modes of learning and focus on basic numeracy skills. Critical thinking for both explicit and infusion maths lessons is emphasised and schools

are encouraged to carry out school-initiated projects and activities to develop creativity.

Problem-solving has been the central focus of Singapore school mathematics curriculum since 1992. The attainment of mathematical problem-solving skills is dependent on five inter-related components – *Concepts, Skills , Processes, Attitudes and Metacognition.* The key features in the program include the development of concepts through meaningful activities, competence in basic skills, mathematical communication through oral work, group discussion and presentation, investigative work and mathematical thinking. In a study (Yeap 1999) done on a small sample of Singapore secondary school students on the role of metacognition in mathematical problem-solving, it was found that these students solve problems differently in the classroom. Five types of metacognitive behaviours were identified. These are stating a plan of action: clarifying task requirements: reviewing progress: recognizing errors and detecting new development. Very few students were able to assess the difficulty of their tasks. At the primary school, metacognitive awareness is exhibited in a totally different way.

Current research indicates that traditional views on teaching mathematics based on Piagetian theory are being challenged. The concept of conservation has had a great impact on the learning of mathematics for many years in the 60s and 70s. The teaching of numbers in the early childhood classroom was heavily dominated by activities such as matching, sorting and classifying. According to Piaget, as children develop they build up their knowledge personally and independently explore their physical and social worlds. This constructive process is located within and governed by the individual child. However, if this way of understanding is similar across all children, they probably have the same cognitive process to interpret their experiences.

Vygotsky and Scaffolding

Vygotsky viewed thinking as a profoundly social process. Social experiences shape the child's interpretation of the world and language plays an important role as a primary means of communication. Anthropological studies from various cultures have pointed out that humans are inherently social and communicative beings. For example, many children become remarkably skilled conversationalists by 2 to 3 years of age. Today developmental psychologists and educators believe that the social and cognitive are essential aspects of one another. In the classrooms, teachers are trying to adjust learning experiences so as to acknowledge the productive use of the social experiences of the child. Questioning, play activities and collaborative work are embedded in intellectual tasks.

In Singapore, teachers are highly interactive in the classrooms and concentrate a lot on question-asking since questions are a particularly important communication tool. Many of them try to induce key mathematical abilities and skills through social experience. It was discovered that difficult mathematical tasks could be performed effectively by having peer groups and age-mates with differing opinions to exchange ideas. What was most helpful would come from the teacher's instructions and questions, providing correct explanations and pointing out errors. According to Vygotsky (1978), through cooperative dialogues with more knowledgeable classmates during challenging tasks, children learn to think and behave in ways that reflect their community's culture. Vygotsky believed that as more mature partners – both adults and peers – offer guidance to children mastering culturally meaningful activities, the communication with these partners becomes part of children's thinking. (P 19, Berk & Winsler, 1995).

Vygotsky believed that it is not so much WHO participates in the social exchanges, adult-child or child-child, as HOW these children participate in their collaborative activity that is so significant. Rogoff (1990) has identified 'guided participation' a way of social experience as most effective in stimulating children's cognitive growth. Another way is scaffolding (Wood 1989) and this is a metaphor used to refer to a support system for children's efforts and which is sensitive to their needs. Adults prompt and monitor children's learning in this social engagement and encouragement. This is a region in which a transfer of ability from the shared environment to the individual takes place and is called the zone of proximal development, a most well known concept in Vygotsky's work in cognitive development of the young.

Scaffolding refers to a special quality of adult-child interaction or collaboration. Berk and Winsler (1995) identify five key components and goals in effective scaffolding which can be applied to maths learning and teaching. These are

1. Joint problem solving
2. Inter-subjectivity
3. Warmth and responsiveness
4. Keeping the child in the ZPD (Zone of Proximal Development)
5. Promoting self-regulation.

Briefly, the first component of scaffolding engages the child in an interesting and culturally meaningful, collaborative problem-solving activity. (adult-child or child-child groupings.) Inter-subjectivity refers to the process whereby two participants who begin a task with a different understanding arrive at a shared understanding. Warmth and responsiveness concern the emotional tone of the interaction and it helps when the adult is pleasant and gives verbal praise. To keep the child in the ZPD zone means that the adult structures the task in such a way that it is appropriate and challenging. Another goal of scaffolding is to foster self-regulation and this includes letting the child to make decisions and to solve the problem himself.

What makes effective 'scaffolding' varies from culture to culture. Its characteristics can only be understood in terms of the values and requirements of the child's

society as a whole. According to Berk and Winsler (1995) Vygotskian scaffolding is limited to Western culture children and some other societies may have different but socially appropriate ways of interacting with their young. However, ZPD is a specially useful framework for mathematical school learning. Vygotsky observed that effective teachers plan and carry out learning activities within children's ZPDs, through dialogue and scaffolding.

Feuerstein and Mediated Learning Experience (MLE)

How do young children make the jump from a natural level of mental organization to higher cognitive processes with the assistance of their caregivers or teachers ? It is found that if the child and the adult focus their attention together, the development of higher order thinking is mutual. Research shows that this kind of joint attentional focus e.g. between mothers and babies provides a communicative context whereby language and problem solving are enhanced. Mothers provide short, clear explanation of what the baby is looking at, long before the infant is able to verbalise what is seen.

The theory of mediated learning experience (MLE) dates back to the 1950s. Reuven Feuerstein (1980) developed it to explain individuals' different propensities for learning referring to an example of young adults emigrating from different cultures to Israel showing different levels of learning propensity in adapting to Israel's technology-oriented society. Some of these variations are explained by the nature of the cultures from which these individuals came. What is more interesting, according to Reuven are the differences in the learning propensities among individuals belonging to the same culture. In this respect, the observed intra-group differences were often greater than the inter-group ones. Observations made by researchers attempting to define and explain the cognitive structure of culturally different groups could not pin point it to the culture the immigrants came from. (Skuy, 1996)

Feuerstein links the differences in learning propensity to an individual's exposure through MLE to his own culture, irrespective of its nature or level of conceptualisation, technology, or institutionalized education. Culturally different individuals have become "different" by learning their own culture. This learning experience, usually gained through an MLE process, turns individuals into efficient learners. They use their previously acquired learning experiences to confront a new culture. Culturally deprived individuals, on the other hand, have not been exposed to their own culture. They have not learned to learn and hence it is difficult for them to adapt to the new, more complex conditions of life. Therefore, according to Reuven cultural difference is not the same as cultural deprivation which is a universal phenomena and can be observed in a large variety of ethnic, socio-economical and professional environments. Cultural deprivation or lack of MLE lowers the individual's 'flexibility and elasticity'.

Feuerstein has shown that individuals have the potential to change and are modifiable if provided with the opportunities to engage in the right kind of interaction. The quality of this interaction is paramount in determining and allowing the individual to develop efficient thinking skills that will enable him or her to become a self-regulated learner. Embedded in MLE is a process by which a mediator organises and interprets the world to the child. When an individual gives meaning to events, helps children select relevant from irrelevant variables, assists in abstracting rules for regularly occurring phenomena, and generally attempts to develop children's abilities to think, that individual is engaged in mediated learning.

MLE begins within the family context with parents and significant others passing on cultural norms, values and modes of thought from one generation to another. A lack of MLE leads to deficient cognitive functioning and low levels of modifiability. The child fails to adapt to and learn from interactions in his/her environment. Many classroom problems in learning are the result of insufficient or inadequate mediated learning experience.

Examples of Scaffolding and MLE

The following two lessons based on Feuerstein's work with MLE are taken from Rodriguez and Bellanca's (1996) examples illustrating mathematical ideas and topics which work well with young children. The use of concept themes and problem solving are infused into the maths instruction.

SAMPLE LESSON *(Example 1)*

COWS and CHICKENS

Problem: How to solve a math problem.
Focus Intelligence: Logical/Mathematical
Supporting Intelligences: Verbal/Linguistic, Visual/Spatial and Interpersonal

CHECKING PRIOR KNOWLEDGE

1 Ask the class to tell you what they know about cows and chickens.

2 Sketch each animal as they reply.

STRUCTURING THE TASK

1 Write on the board: There are four cows and three chickens. How many feet and tails are there all together?

2 Put students into pairs and give each pair one pencil and one piece of paper. Invite them to agree on one answer to the question. They can do work on the paper to figure out the answer. Invite them to make their notations large enough for others to see from the class circle.

3 Check for understanding of the task.
4 Circulate among the pairs and observe how they work together to solve the problem.

LOOKING BACK

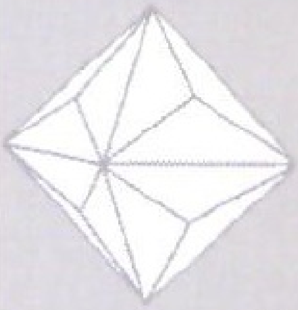
1 Assemble the pairs in the class circle. Ask a number of the pairs to share what they did to solve the problem. Ask them to save their answers. Invite all to use their listening skills when they are not speaking.

2 Comment on each strategy with positive feedback.

BRIDGING FORWARD

1 Ask random pairs to tell what they learned about problem-solving

2 Identify the answers.



It is important not only for students to obtain information, especially when that information has connection to their prior knowledge and experience, but also for students to use that information. The more stimulating and creative the opportunity for use, the more likely it is that students will "make sense" of the raw information. By transforming print information into another medium, the students will have a richer opportunity to develop a second or third intelligence, lock the information into short term-term memory, and build a sense of pride in their work. In essence , as Feuerstein points out, the teacher structures a meaningful task. In addition, the task provides multiple opportunities for the teacher-mediator to mediate for meaning as the project underfolds.

This primary-grade mathematics task can be adapted by the teacher to fit into the child's cultural experience.

SAMPLE LESSON *(Example 2)*

SHAPES

Problem: How to describe shapes in the world around us.
Focus Intelligence: Naturalist
Supporting Intelligences: Logical/Mathematical, Visual/Spatial, Interpersonal and Bodily/Kinesthetic

CHECKING PRIOR KNOWLEDGE

On the board, draw a circle, a square, a rectangle, and a triangle. Ask each student to think where he or she might have seen these shapes. Allow pairs to discuss the sightings before you ask individuals to share. Under each shape, list the appropriate responses.

STRUCTURING THE TASK

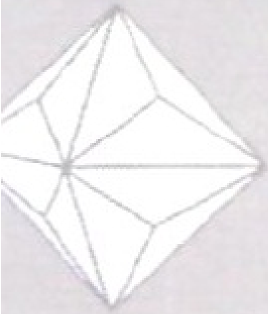
1 Break the class into four to six groups. Invite each group to form the shape you specify.

2 Invite the class to label each shape as you point to each example on the board. Ask "What makes this shape special?"

3 Give each group a worksheet with the four shapes. The groups will search throughout the room to find objects in which each shape is found. Let them write the object's name or sketch it.

4 Conduct a round robin and invite explanations/reasons for each selection.

LOOKING BACK



Invite each group to add an example to each list on the board. Conduct a round robin until all lists are complete.

BRIDGING FORWARD

Instruct each child to take a shape worksheet home so that they can find at least three items which contain each shape. Use the reciprocal model to check for understanding.

Assessing Student Performance

Each child can identify and name the shape within an object and can explain why each example is reflective of the definition.

Variation

Give each child a page of shapes to cut out and make a simple picture. Ask the child to explain why the picture is a specific shape.

Recognizing Patterns

For the primary grades, learning about mathematical shapes is an important topic. Many workbooks end the lesson by asking students to match a shape with a word. The above lesson designed to mediate transcendence goes beyond simple recognition of isolated shapes. It instead enables children to recognize each shape wherever it may be located. Notice how the lesson calls for explanations of the definition .

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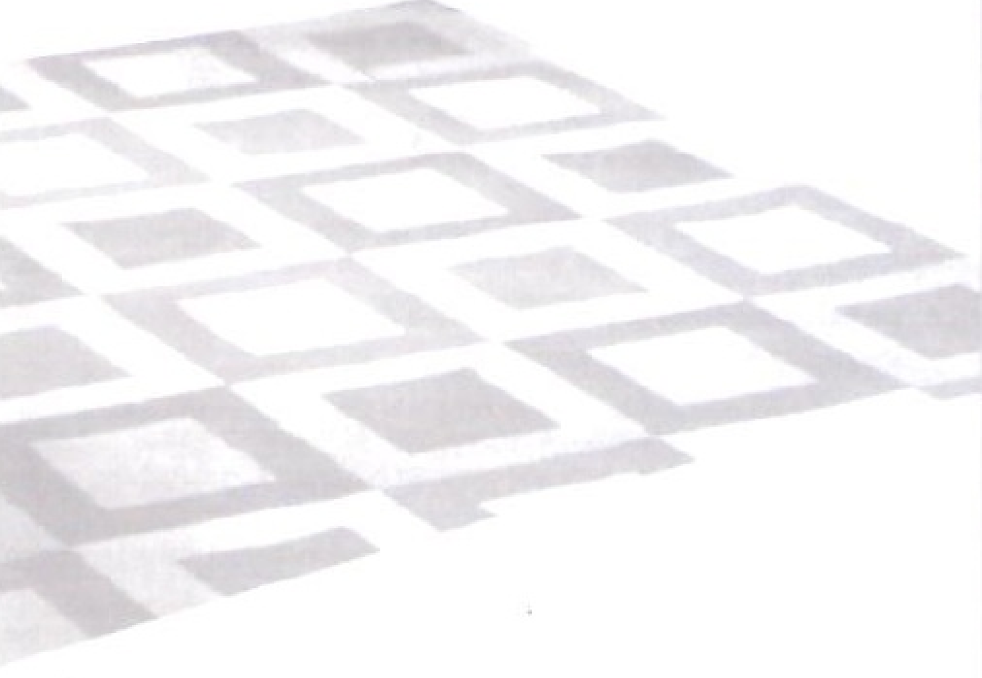


Infusing National Education in Mathematics in Singapore

Teh Keng Seng

To realise the "Thinking School, Learning Nation" vision launched by Prime Minister Goh Chok Tong in 1997, MOE has initiated and has placed emphasis on Information Technology (IT), National Education (NE) and Thinking Skills in Singapore schools.

NE was introduced to make our pupils understand Singapore, its limitations and vulnerabilities, its goals and ideals and why certain things were done. It aims to involve students in the things that concern the society and ultimately themselves. Infusing NE in mathematics can be meaningful and teachers can demonstrate the relevance of mathematics in everyday life to pupils by selecting relevant and current topics in their examples and exercises. Teachers can use real life contexts as a background for the teaching of social responsibility in making choices and also the teaching of good citizenry.

To infuse NE successfully in the curriculum, the teachers must themselves believe in them. Only then, will the pupils feel the enthusiasm and commitment demonstrated by them. The six NE messages are listed below:

1. **Singapore is our homeland; this is where we belong.** We want to keep our heritage and our way of life.
2. **We must preserve racial and religious harmony.** Though of many races, religions, languages and cultures, we pursue one destiny.
3. **We must uphold meritocracy and incorruptibility.** This means opportunity for all, according to their ability and effort.
4. **No one owes Singapore a living.** We must find our own way to survive and prosper.
5. **We must ourselves defend Singapore.** No one else is responsible for our security and well-being.
6. **We have confidence in our future.** United, determined and well-prepared, we shall build a bright future for ourselves.

The following pointers are useful when teaching NE in mathematics:

(a) Be natural, present the facts and comment on relevant points to bring out the NE message.

(b) Do not preach and there is no necessity to mention the fact that we are going to do a NE lesson.

(c) Best done when the question is in the textbook and when pupils ask related questions, teachers can then elaborate on the points raised.

(d) Just get pupils to be aware of the topic. If pupils have more questions, the teacher can elaborate.

(e) Use situations that pupils are familiar with like, water usage, National Service, Certificate of Entitlement (COE) and electronic road pricing (ERP). Ask related questions and initiate a discussion if there are pupils interested in the topic. For example, when dealing with traffic conditions, ask them if they know of traffic situations in Bangkok or Kuala Lumpur. What effect will it have on Singapore if we have to declare a public holiday on the days when international meetings are to be held here because we have to ensure smooth traffic? Why are we exploring the possibility of building desalination plants in a big way?

The following are some of the mathematics questions phrased using figures from newspaper reports so as to bring out NE messages. These questions will allow teachers the opportunities to elaborate on the social topics involved. Some of these problems are suitable for Upper Secondary classes like problems involving logarithms, permutations and combinations. For the Lower-secondary classes, problems involving arithmetic such as percentages will be very suitable.

Example 1

In January 1999, a Buddhist monk donated $100 000 to the Muslim Missionary Society Singapore (Jamiyah) towards the building of its new nursing home. If Jamiyah has to raise $1.7 million for the project, express $100 000 as a percentage of $1.7 million, giving your answer correct to 2 decimal places.

Example 2

In November 1999 the Ba'alwi Mosque distributed part of the 45 tonnes of dates donated by the Saudi Arabia government to 20 non-Muslim charity organisations in Singapore and the general public of all races and religion on a first come, first served, basis. If 26 tonnes are for the public and each person is given 350 gm, how many people received the dates from the mosque?

These two questions are specially for infusing the second NE message that: **We must preserve racial and religious harmony. Though many races, religions, languages and cultures, we pursue one destiny.**

The teacher only need to mention: Notice that, a Buddhist monk is donating money to a Muslim organisation. This is a good example of religious tolerance and co-existence. With the picture and newspaper report on Microsoft Powerpoints, the message will be very effective. (Refer to Power Point presentation). Example 2 shows a Muslim organization also doing its part in practising racial and religious tolerance. If your class has pupils from different races and religion, you may ask them to elaborate on how they interact among themselves and what they learnt by interacting among themselves. Encourage them to learn good points from one another.

Example 3

The Singapore Assault Rifle 21 (SAR 21) developed locally will be the standard issue for Singapore soldiers in the 21st Century, replacing the M-16. The SAR 21 boasts many advance features such as a laser aiming device. The overall length of the SAR 21 is 80.5 cm while the M-16 has an overall length of 99 cm. The weight of a SAR 21 is 3.98 kg while the M-16 has a weight of 4.06 kg. Calculate, giving your answers correct to 2 decimal places,

(a) *the percentage difference in the overall length of the SAR 21 as compared to the M-16,*

(b) *the difference in the weight of M-16 as compared with the SAR 21.*

Mindef goes unde

Example 4

Example 4

The Defence Ministry (Mindef) started the construction of an ammunition storage complex deep underground in a disused quarry in Mandai in 1999. The project will help to save more than 300 ha. of land for other valuable use in land scarce Singapore. If 1 square metre of land is valued at $2580, calculate the cost of a piece of land of area 300 ha.

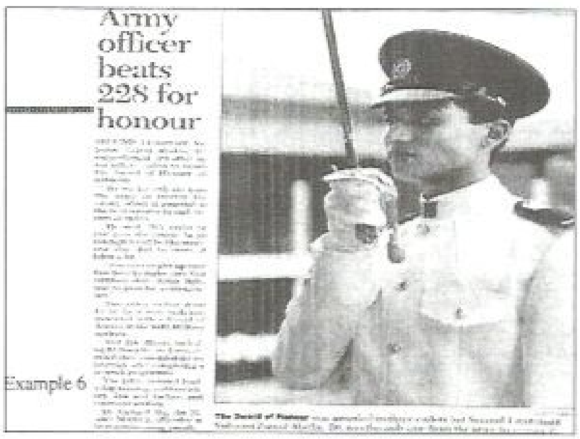
These two questions deal with the Singapore's Armed Forces. Example 3 gives the opportunity for teachers to mention that we, as a small nation, need to defend ourselves with our limited manpower. Hardware improvement and upgrading is an important criterion for our Armed Forces. The NE message that we must ourselves defend Singapore could be delivered using both examples. In addition Example 4 can also be used to deliver the message that, as Singapore has limited land, we need to constantly look out and find ways to maximize our land use. The NE message that: **No one owes Singapore a living. We must find our own way to survive and prosper** is also clearly demonstrated here.

Example 5

The Singapore government gave out $35 million to 121 000 students in the form of Edusave scholarships and bursaries in 1999. Calculate the average amount received by each student giving your answer correct to the nearest 50 cents.

Example 6

Second Lieutenant Suhaimi Zainul Abidin was the only officer from the army to receive the Sword of Honour at the Commissioning Parade in April 1999. Suppose he and the other two recipients of Sword



Army officer beats 228 for honour

Example 6

of Honour from the Air Force were asked to take a photograph with the Minister officiating at the function, how many different arrangements are possible for a group photograph of the four standing in a row?

Examples 5 and 6 best delivers the NE message that: **We must uphold meritocracy and incorruptibility. This means opportunity for all, according to their ability and effort.** The Edusave scholarships and bursaries had been launched to encourage pupils to do their best. It is given out to pupils who perform well in their studies and to those who had shown great improvement in their studies. The teacher may ask the pupils how many of them got the scholarship or bursary last year and take the opportunity to bring home the point that those who work hard will be rewarded irrespective of your race and religion. It is one of the best examples to illustrate where the practice of meritocracy and incorruptibility are observed in Singapore.

Example 6 is suitable for use in the teaching of permutation and combination. There are two NE messages here: (i) We must uphold meritocracy and incorruptibility and (ii) We must, ourselves, defend Singapore.

Example 7

The vehicle population of Singapore at the beginning of 2000 was estimated to be 560 000. With the effective implementation of Certificate of Entitlement (COE) and Electronic Road Pricing (ERP), the vehicle population is projected to grow at an annual rate of 3% so that after a period of t years the new vehicle population is $560\ 000(1.03)t$ *. Find the year in which the vehicle population would reach three quarter of a million.*

Example 8

In January 1999, Mr Sim invested $10 000 in the bond issued by HDB which promises a return of 3.75% per annum such that at the end of n years the amount he will get is given by $\$10\ 000(1.0375)^n$*. Find*

(a) *the amount Mr Sim will get in January 2004.*

(b) *the year in which the amount will just exceed $13 500.*

The context of Example 7 is something that most Singaporean pupils are familiar with. Thus, the abstract equation becomes more concrete and pupils can visualise in their minds using concrete images of the question. This makes a meaningful connection with their schema. Using this example, teachers can further encourage pupils to consider the implications of heavy traffic jams and the ensuing waste of time and drain on our economy. This discussion will lead pupils to reflect on the personal responsibilities and consider their part in national issues. Singapore is the first country in the world to implement the COE and ERP system. It is a necessary evil and perhaps the best systems for use in our land scarce nation. The teacher can also get pupils to suggest any good ideas that can satisfy the public who longs to own cars. More roads can be built but it could require the government to acquire private lands for these purposes. How would you react if your house or flats were to be acquired for building roads? How can we strike a balance? The NE message for the question is: **No one owes Singapore a living. We must find our own way to survive and prosper.**

Example 8 is used in the teaching of logarithms in Additional Mathematics. As Singapore is a country with no natural resources we need to create opportunities for our survival. The opening up of the financial sector and creating bond markets are examples of what the government is doing to find ways for our survival. The NE message is none other than: **No one owes Singapore a living. We must find our own way to survive and prosper.**

Example 9

The government is planning to spend $4.4 billion over 7 years to upgrade and rebuild schools in Singapore under the Programme for Rebuilding and Improving Existing Schools (Prime). Each school will have bigger classrooms, from the present 64 square metres to 90 square metres to accommodate more computers. Calculate the percentage increase in the floor area of the classroom.

Congratulations, the separation's a

Success

We have the right people, a bunch of people in there who are among the best in the world... I have no question about submitting my head to them.

STREATS 11-4-2001

Example 10

Example 10

Medical history was made in Singapore in April 2001 when a team of Singapore doctors performed an operation to separate a pair of Siamese twins from Nepal, born joined at the head. The marathon operation started on Friday at 5 pm and it lasted till 4.20 pm on Tuesday. Calculate the total time taken to perform the operation.

Example 9 is something that many pupils are familiar with. They may be in the thick of it now, busily preparing to relocate to a new location while their school is undergoing upgrading or rebuilding. The government is preparing its citizens to face the future with confidence by upgrading the education level of its people.

Many Singaporeans were proud that the high profile operation, carried out by the team of Singapore doctors, was a success. It puts Singapore on the world map of countries with high quality health care and top specialists.

The NE message for the two questions is: **We have confidence in our future. United, determined and well-prepared, we shall build a bright future for ourselves.**

The following questions were constructed so as to facilitate the discussion of certain social issues with the pupils. Some of them will act as encouraging pointers for students to work hard while others hope to get pupils to preserve the social fabric of the society. The NE message for most of the following questions is: **Singapore is our homeland; this is where we belong. We want to keep our heritage and our way of life.**

Example 11

In 1996, 118 young girls and 119 young boys were infected with sexually-transmitted diseases (STDs). In 1999, 177 girls contracted the STDs.

(a) Calculate the percentage increase in the number of young girls contracting STDs from 1996 to 1999.

(b) If the number of boys contracting the STDs in 1999 represents a 23.5% drop as compared with 1996, find the number of boys contracting STDs in 1999.

Example 12

In 1999 there were 1080 Singaporeans infected with the Aids virus, 150 more than in 1998. Of these, 11 were babies who caught the virus from their mothers, 126 were women and 88 of these women were married. Calculate the

(a) percentage increase in the number of Aids cases from 1998 to 1999

(b) percentage of male Aids carriers in 1999

(c) percentage of women Aids carriers who were married.

HOW IT'S TRANSMITTED

More babies getting HIV from mothers

Number has shot up to 11, from only one in 1991; this could be due to more married women being infected

Example 12

These two examples provide teachers an avenue to talk about the health problems in Singapore. We must be aware of other social problems in our society. Teachers can take the opportunity to caution the pupils to be careful when they deal with boy-girl relationships and to learn to take care of themselves.

Example 13

The Singapore Power invested $12.5 million for a 42.5% stake in a desalination company in 1999. The company hopes to build desalination plants to provide an alternative source of water to Singapore.

Another Singapore related company took a 5% stake in the company.

Find the amount invested by the other Singapore company.

THE STRAITS TIMES

WASTE NOT, WANT NOT

- Brushing your teeth with the tap running wastes 44.5 litres, which can fill a child's water bottle for 10 weeks.
- Letting the shower run while soaping wastes 60 litres, which is all an average adult needs for a month.
- Washing dishes under a running tap wastes 155 litres. This can fill the water bottles of a primary school class for a week.
- Washing vegetables under a running tap wastes 48 litres, which is what a runner needs to run 13 marathons.

Example 14

Example 14

Brushing your teeth with the tap running wastes 44.5 litres of water per brush. How much water was wasted by a boy who has the habit of brushing his teeth twice a day with the tap running for a year with 366 days? How much more money has his parents to pay per year for his wasteful habit if each litre of water costs 0.112 cents?

Examples 13 and 14 will give teachers an opportunity to deliver the message that Water is precious, do not waste the precious commodity. We can also mention the reasons for Singapore to look for an alternative water source as we cannot depend only on Malaysia for our supply of water. They may need to keep the water for their own use in future and not be able to supply us. Our people must be prepared for the day when we have to drink water that is recycled, or from desalination plants which may not taste quite the same as what we are used to having.

Example 15

Due to the economic crises in the late 1990's the average household income of the bottom 10% of our society had dropped from $258 in 1998 to $133 in 1999 while the average household income of the top 10% increased from $x in 1998 to $15 451 in 1999.

(a) *Find the percentage difference in the income level of the bottom 10% of our society from 1998 to 1999.*

(b) *If the top 10% earners in our society increased their earnings by 2.67% find x, giving your answer as a whole number.*

Example 16 ST 10-2-2001

Example 16

The median family income of Singapore families increases from $3 080 in 1990 to $4 940 in 2000. The average incomes of households with graduates rose from $7 118 in 1990 to $9 827 in 2000 while the average household income of families without secondary education rose from $1 504 in 1990 to $1 667 in 2000. Calculate the percentage increase in the household incomes of

(a) *the median Singapore family*

(b) *families with graduates in the household*

(c) *families without secondary-education, from year 1990 to 2000.*

The figures provided by these 2 problems clearly show the pupils that if they want to have a better life, education upgrading is one good route. We need to constantly upgrade the educational ladder of our people. These examples can also be used to urge and spur the disinterested pupils to work hard to achieve greater heights in their future.

Example 17

The crime rate in Singapore reached a record low of 39 143 cases in 1999. This represents a 21% drop as compared with 1998. Find the number of crime cases recorded in 1998. The crime rate in Singapore is 1 005 per 100 000 in 1999. If the population of Singapore in 1999 is 3.65 million, how many cases of crimes were committed in 1999?

Teachers can talk about the low crime rate in our nation and how we can play a part to keep it this way and further improve on it. The NE message that **Singapore is our homeland; this is where we belong. We want to keep our heritage and our way of life.**

Example 18

The table below gives the index to measure the national pride of the citizens of some countries.

Country	Austria	Japan	Ireland	Singapore	The Philippines	USA	Canada
Index	17.6	16.4	16.3	17.2	16.5	17.2	16.6

Represent the above information using a bar chart. Hence, state among the countries listed above

(a) the country that has the highest index for national pride

(b) which countries have the same index for national pride.

The NE message is: **Singapore is our homeland; this is where we belong. We want to keep our heritage and our way of life.** The teacher can ask pupils what they like about Singapore, what makes them proud to be Singaporeans and what they think we can do to make Singapore a better place.

Example 19

Meng Kuang has saved a small sum of money from his weekly pocket money. After receiving a total of $108 as "ang pow" money from his relatives during the Chinese New Year, he decided to donate one-fifth of his money to the Community Chest of Singapore.

His other siblings also donated a total of $148 to the Community Chest. If their total contribution to the Community Chest is $200, how much did Meng Kuang have saved, originally?

Singapore does not adopt the Western welfare policy. It encourages self-help among its people. The teacher can ask their pupils how many of them have done community service and donated to charitable causes this year? If they have the ability, would they like to contribute to the underprivileged in our society?

Example 20

During the Inaugural World Trade Organization meeting held in Singapore in 1997, five ministers from Europe arrived and there were only five top hotels to accommodate them. If each hotel can only take in one minister, how many different ways can we accommodate the five ministers?

This question is suitable in the teaching of permutation and combination. The successful conclusion of the Inaugural WTO meeting in Singapore had boosted our standing on the world stage.

Conclusion

I have used most of these questions in the course of teaching my mathematics classes and found them to be useful. The pupils were made more aware of the

social situations in Singapore and on several occasions I had the opportunity to discuss more on the topics raised by the pupils, relating to the questions posed. I have also shared my teaching methods on NE through Teachers' Network and had received positive feedback. If more teachers are aware of this approach, it will greatly help to realise the vision of TSLN. Teachers can also construct fresh mathematical questions on NE if they are on the lookout for more ideas from the daily newspapers or periodicals. These questions can then be shared among fellow teachers, or through the Teachers' Network. Do not be discouraged if suitable NE related stories do not come easily in our papers. With perseverance we will be able to spot related articles or reports that can be used to phase-in to mathematical questions.

The strategies to infuse National Education in mathematics, to link learning to real life, encouraging pupils to think and discuss current social issues will make the learning of mathematics fun and meaningful for the pupils.

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A Mathematical Teaching Model

Douglas Edge

More years ago than I care to admit I was a graduate student at the University of Maryland when professors Robert Ashlock, John Wilson and Martin Johnson were developing a teaching model designed to help teachers better plan for instruction. They believed that when we teach children mathematics, we do so with the intent that the children will understand the mathematics they learn, will recognize important uses of mathematics, and will not forget what they learn too quickly (Ashlock, Johnson, Wilson and Jones, 1983). However, they were concerned that planning for instruction be based on sound theory. They stressed the importance of connecting content instructional objectives with the related sets of learning activities.

The Model

The model that Ashlock et. al. (1983) developed comprised of six parts which they called "activity types". The first three of these parts focused on types of activities where teachers planned for children's conceptual understanding, ranging from introductory ideas to complex pattern seeking behaviours. The remaining three model parts included activities for drill and recall, problem solving and applications, and, at the core, assessment. Subsequently I adapted their model for my own use with both pre- and in-service teachers. It is this model I wish to describe in this article. From Figure 1 you may notice that I have essentially collapsed their initial three activity types into one, which I label 'understanding'.

Figure 1: Teaching Model with its four activity types

Application of the Teaching Model

When using the Teaching Model to plan for instruction, each of the parts of the model has it's own purpose and therefore it's own set of typical activities. Let us consider each part in turn, beginning with 'understanding'.

Understanding

The purpose of this part of the model is threefold:

(1) To introduce pupils to the concept, let them develop or abstract a general understanding of the concept, and find patterns or relationships within the concept. Consider for example, a lesson on 'number bonds' with combinations to six, assuming prior knowledge with combinations to five. A teacher might give pupils a set of six blocks or counters and ask them to divide them into groups. To help the pupils with the activity, the children might be given mats (perhaps cut from Vanguard), such as those in Figure 2, to assist them. The teacher would then call upon the children to discuss their stories. Ashlock et. al. (1983) would label this kind of activity as "**initiating**" as children are building new knowledge based on existing structures.

(2a) (2b)

Figures 2a and 2b: Sample mats for showing number bonds, in this case, 4 + 2 = 6.

(2) Next, the teacher might capitalize on their pupils 'play' and ask them how they would record their number stories. Using the mat again (shown in figure 2a), the teacher could then ask pupils to tell what stories they have shown and ask them how they wish to write the corresponding number sentence. With the second mat (shown in figure 2b), pupils might begin by writing the number sentence, then combine the blocks in the lower portion, before finally writing the answer. The same mat could be used later for subtraction, of course. In this case the children might begin with the long portion at the top with the 6 blocks. After asking the children to slide 4 of the blocks to one of the lower sections, the teacher would ask, "How many are left to move to the other section?" During these activities pupils, as well as developing understanding of the concept, key vocabulary and conventional ways of writing the number sentences would be introduced. Teachers later would extend these activities using other materials such as Cuisenaire Rods and number lines. Ashlock et. al. (1983) called these middle level understanding activities "**abstracting**".

(3) Once the children have developed some understanding of the concept, the teacher might then plan additional work to focus on inter-relationships within the concept and between existing concepts. For example, and perhaps using the mats once again, or the Cuisenaire Rods or number lines, pupils might be asked to find all the number bonds that add to six and write them in some kind of order. The teacher might ask, "How do you know you didn't miss any?" Ideally, children could point to their diagrams and answer by saying they started with one group of 6 and wrote "6 + 0 = 6", then they moved one block from the left to the right side of the mat getting another bond and wrote "5 + 1 = 6". "We continued moving one at a time until we got all the way to 0 + 6 = 6."

A different task altogether but one still requiring children to relate known facts might be to ask them to focus on bond families such as 2 + 4 = 6, 4 + 2 = 6, 6 – 4 = 2, and 6 – 2 = 4. These understanding activities where children are asked relate such information were labeled in Ashlock et. al. (1983) original Activity Type Cycle as "**schematizing**".

Consolidating

Once teachers are confident that their pupils have an understanding of the basic concept(s), they may plan activities characteristic of the next part of the model. The primary purpose when consolidating is to help children recall with reasonable speed and accuracy the facts or skills associated with the concept. Children are no longer asked to explain why something is true, or why it works. Routine drill activities, including the use of flash cards can now be used. Some teachers may develop simple games or create situations where children can practice their skills. The following kinds of games all become part of an experienced teachers standard repertoire: "Climb the ladder" with number bonds written on each rung (children answer 'going up the ladder' as quickly as they can), "Beat the Bounce" with the teacher dropping a ball and saying a number bond at the same time (children are expected to answer before the ball falls to the ground), and "Buzz" (children count and if the number is a product of some predetermined number they must say 'buzz' rather than the number; for example, if practicing the 6 times tables, one would count ...4, 5, buzz, 7, 8, ...11, buzz, 13, 14, ...).

Reinforcing and helping pupils keep track of their progress is also an important consolidating activity. Teachers, for example, may ask pupils to prepare bar graphs to show their improvement with their recall of number bonds. A teacher might select ten bonds (a few easy, a few difficult) from a set of flash cards. Children are shown these cards in succession, say on a Monday for 5 seconds only, and asked to record the number of correct answers on a bar graph. Using the same set of flash cards, but in a different order, this activity is repeated for several days in a row. Hopefully, by the end of the week children will see their own progress. Figure 3 highlights an optimistic result!

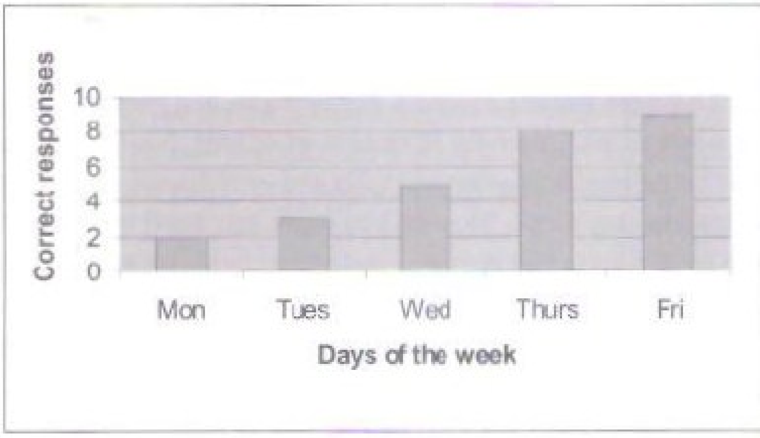

Figure 3: Class Progress Graph

Transferring

When planning activities with this portion of the model in mind it is assumed that children now have good understanding of the concept(s) and have reasonably accurate recall of, and skill with, those concepts. The focus thus shifts to preparing tasks for the pupils where they can apply their knowledge to new situations. Typical tasks will involve solving routine and non-routine problems including using all the heuristics we have become familiar from the work of George Polya (1973). Students may play games that require some element of strategy or study topics that promote special interest in mathematics. They may be assigned special investigations or projects. Certainly they would be expected to use their new found skills to relate to everyday uses of mathematics.

Occasionally, the line between what constitutes applying new knowledge (transferring) and developing early understanding of new concepts (understanding) becomes somewhat blurred. For example, if pupils are asked to use protractors to investigate relationships among angles of a triangle, the activity may be seen as 'transferring' in that children are now applying their protractor skill abilities, yet the activity may also be seen as a concept formation activity relating to triangle / angle relationships. In my graduate student days, we classified such activities as T/I (transferring / initiating).

Assessing

The model is not donut shaped. There is no 'hole' in the middle. Assessment must be understood as being central and integral to the teaching process. Occasionally, assessment is formal in the sense of using end-of-term tests and examinations such as the Primary School Leaving Examinations (PSLE) at the end of Primary six, or may be informal in the sense of gaining information from children's class work, homework, responses in class or daily quizzes. A third form of assessment also critical to the learning process is that of diagnostic assessment. Diagnostic assessment may or may not utilize formal procedures. A teacher may find that he or she has designed a lesson primarily to promote recall of facts (consolidating) but as the lesson progresses the teacher begins to suspect that the children have a few conceptual difficulties (assessing / diagnosing). The teacher then decides "to revert to an earlier part of the model" (understanding). All our understanding of Bloom's taxonomy (Bloom et. al. 1956) and our Singapore adaptation using the knowledge, comprehension and application (K, C, A) levels of understanding (Ministry of Education, 1996) fit directly into this part of the Teaching Model.

Model Summary

Let's take a moment to review the model with another set of concepts in mind: Place value. To promote the various levels of understanding a teacher, using base ten blocks, may give students a pile of the small cubes and ask them to tell how many there might be. After asking, "Is there an easy way to keep track of how many blocks we have counted?", the children might be encouraged to put blocks into piles of equal sizes and count by groups (early understanding).

The teacher might then introduce the 'tens-block' by showing the children how ten cubes is equivalent to one ten's block and explaining that they are the same in that they are equivalent amounts, and different in that one is one set of ten, the other ten sets of one. The children would be asked to count again using the base ten's blocks and could now model 28 using 2 tens and 8 ones, or 364 using 3 hundreds, 6 tens and 4 ones. They could tell you about the number, draw a picture of the number or write the number and be able to explain that the number was comprised of groups of ten and ones. (Middle understanding or abstracting)

Next, the teacher might ask children to model both 56 and 65. The teacher would ask the children to say what the 5 in 56 and what the 5 in 65 means. The teacher might also ask children to model, discuss and compare the numbers 402, 42, and 240, involving not only place value concerns but also the difficulties associated with zero-as-a-place-holder (Late understanding or schematizing).

For an example of a 'consolidating' place value activity, consider the following game: Children play in pairs. Each child is given three dollars in the form of three $1 coins (or 'three hundreds' of base ten blocks). Children take turns rolling a pair of dice and multiplying the two numbers together. The first player must give the opponent the amount rolled, say 4 x 6, or 24 cents, necessarily breaking down one of the $1 coins and thus practicing place value. The opponent then takes his or her turn. Play ends when one child either has no money left or after a fixed period of time, say ten minutes, when the winner is determined by who has the most amount of money. For a 'transferring' activity, children may be asked, if using only $1, 10¢ and 1¢ coins, how many ways they can make up $1 or $2. A diagnostic 'assessing' task might involve asking children to count by tens in writing starting at 579. From my own experience, I have observed a number some children write 579, 589, 599, 5109, 5119, ... These children seem to have a sense of pattern but not of traditional place value conventions.

Reflections on the Model

Comparisons to a Mathematics Curriculum Framework

In Singapore all teachers should be familiar with the pentagon shaped framework used to conceptualize the teaching of mathematics in Singapore primary grades (Ministry of Education, 2001). That model, shown in figure four, has as its core, problem solving. We teach children mathematics so that they will be able to solve problems! To facilitate the attainment of this primary objective there are five components to the model: skills, processes, concepts, attitudes and metacognition. How does this Framework compare to the Teaching Model?

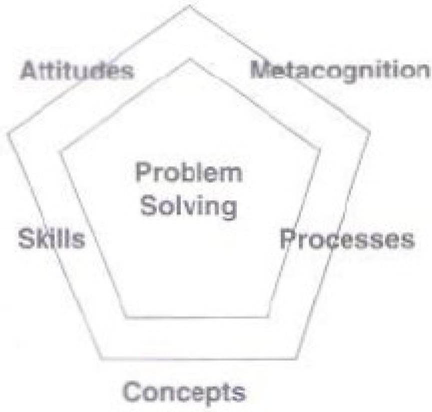

Figure 4: Singapore Mathematics Curriculum Framework

The processes, concepts and skills of the Singapore Framework all would have their origins in the understanding activity types of the Teaching Model. Clearly, problem solving, is integral to the transferring portion of the teaching cycle. 'Metacognition', however, it may be argued, is inherently part of all components. Pupils could be encouraged to ask themselves, "Do I understand the ideas well enough so that I can now start to memorize them?", "Can I do my work fast and accurately enough now, so that I am ready to start applying the rules or concepts?", or "Am I really ready to start learning a new topic?". With the 'attitudes' component of the Framework, one would expect that if pupils understand the work they are doing, have some facility with the procedures and skills involved, and can use their knowledge, they would develop a positive dispositions toward mathematics.

Complexity of the Teaching Model

To this point a reader might assume that application of the model implies that one would begin teaching in the understanding portion then proceed in some orderly 'linear' fashion through the various activity types. The reality of teaching of course is much more complex. Most likely teachers would begin a topic perhaps 'transferring' from some known abilities, then work through various aspects of understanding and begin some consolidation activities. The teacher might then revert to some advanced understanding tasks, before returning once again to more consolidation activities. That is, in the real-life classroom, there is likely to be a considerable amount of "to-ing and fro-ing" going on between the various parts of the model.

Curriculum planners might wish to use the model to make certain types of decisions. At the risk of being a bit controversial, let us consider for a moment the use of calculators in the primary grades. Should we allow the use of calculators in the primary grades? Perhaps we could ask the question, if we were to use them when might we? One way to consider the answer might be in the context of the Teaching Model. It could be argued that we would not permit calculator use when we are planning understanding and consolidating activities, but would recognize their value when children were involved in investigation and selected problem solving activities; that is during the transferring component.

Concluding Statements

When Ashlock et. al. (1983) developed their original Activity Type Cycle, they did so with the intention that teachers understand the need for a connection between the content objectives they are required to teach and the kinds of activities they need to plan to promote effective learning. Today it is still no less important that when teachers are planning for instruction they should ensure that direct, appropriate connections between content and instructional activities exist. Just as we have adopted the "K,C,A" model for use with assessment, perhaps we might adopt some form of a "U, C, T, A" model – one that would guide us in our unit and lesson planning. Informed teaching decisions require some kind of framework. For your consideration and perusal, I have summarized key parts of the Teaching Model in **Table 1**.

Teaching Model Components	
Understanding Purpose: to introduce students to new ideas perhaps using known ideas from previous knowledge (initiate), to develop key ideas within the new concept (abstract), and to interrelate these key ideas within the concept (schematize).	Notes: • Includes use of concrete, and concrete-to-picture-to-symbolic models • Involves teaching new vocabulary, conventions for writing, and so on.
Consolidating Purpose: to help pupils develop reasonable speed and accuracy with the new concepts and skills	Notes: • Incorporates both routine and non-routine drill activities as well as simple games. • Includes notions of reinforcement.
Transferring Purpose: to facilitate the use of the concepts in new situations	Notes: • Incorporates Polya's problem-solving model • Highlights interesting extensions into other areas of mathematics, or even other curriculum areas. • Includes applications to real-life situations.
Assessment Purpose: to infer whether or not pupils have mastered key aspects of the new concept.	Notes: • Assessment is meant to be multi-dimensional; that is, includes both formal and informal strategies. • May include pencil-and-paper testing as well as interview and observational techniques. • Decisions may indicate whether students have attained mastery, partial mastery, or limited mastery of concepts

Table 1: *Teaching Model. Adapted from Ashlock, Johnson, Wilson and Jones (1983).*

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From Hand-holding to Autonomy: Construction of Mathematical Knowledge via Thematic Approach

Ng Hoe Cheong

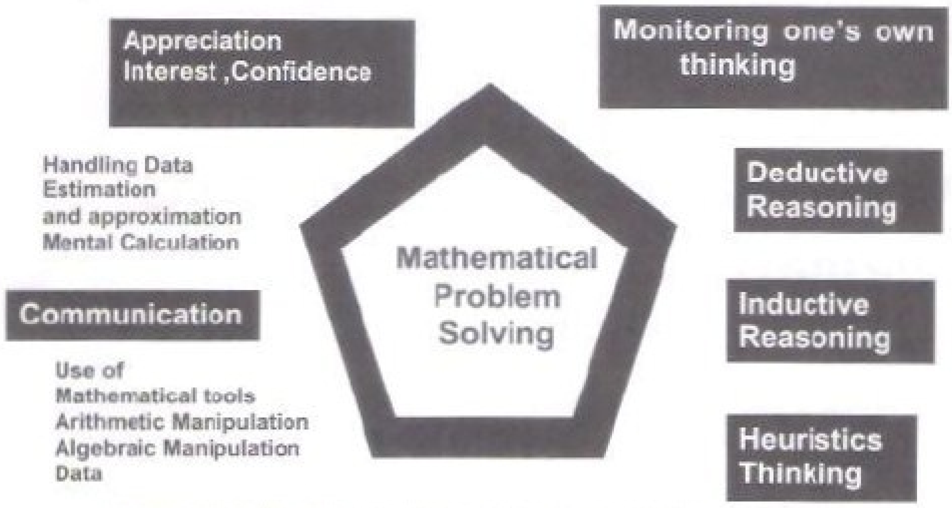
Hand-holding is Important In Early Learning

I recalled that memorable day, when my eight-month old little girl first discovered that she could stand. Holding on to my hand, while she was sitting on the bed, she struggled with much effort to stand on her feet. However, as her leg muscles were still weak, she soon fell backwards and sat down with a bump on the bed. Delighted with the discovery of her new accomplishment, she struggled to her feet again. The process was a tedious one, struggling to get up, falling down, struggling to stand up again but falling, again. Yet, she repeated this tedious endeavour with much enjoyment, delight and determination for a full ten minutes! With a broad smile beaming on her face, she obviously, derived much satisfaction from her new found ability. I offered her my supportive hands, praised, clapped and cheered for her, every time she stood up.

Likewise, in the initial stages of learning mathematics, there is a need to offer the learners support by starting instruction from where the pupils are. There is a need to provide close guidance, give clear and simple instructions, and plenty of encouragement regularly. When the feeling of frustration is minimised, the learner's motivation will be enhanced. The learners will be able to experience the joy of learning. Little successes, as they go through the process, will help the children to develop confidence and competence. The analogy of "**hand holding**" is chosen here because as in any situation, the initial stages of teaching something new should be based on a methodology that provides support.

Ultimate Goal of Education is Autonomy

The Curriculum Planning and Development Division (CPDD) of Singapore provided teachers with the following pentagonal framework for the Primary Mathematics Curriculum:



Framework of Mathematical Curriculum, Mathematics Syllabus, 2001 (p. 10)

I have highlighted some of the essential, yet illusive goals in this framework. We can infer that it is the role of the mathematics educator to equip his pupils with the necessary thinking and communication skills and to impart appropriate affective attitudes which will help them to become **independent**, flexible, creative and effective problem-solvers, leading them towards learner autonomy.

Thematic Learning

This article will explore how mathematics educators can serve their students better by the careful crafting of their lessons via a thematic approach, which allows progressive extension and expansion of mathematical concepts with a natural flow, in a meaningful way. Thematic teaching has been used extensively in many disciplines. According to Caine, R. N. and Caine, G. (1994), *"Themes allow for organisation of seemingly fragmented topics. They are essential tools in the educator's tool kit because they invoke universal ideas and concepts that almost everyone can identify with ..."* [p.118]. A general theme can therefore be harnessed as the central organiser for the subject to be studied. It provides a sense of direction for learning endeavours.

However, the use of themes is rather rare in the teaching and learning of mathematics. It is worth examining this idea, especially in the primary schools. Effective use of themes in mathematics can help to facilitate the establishment of connections among concepts and to build linkages across topics. The building blocks of thematic teaching are fundamental concepts and basic principles which, when understood, will enable the problem-solver to adapt their understanding to tackle new and novel problems. The need to rely on memorised rules and formulae will hence be reduced, as we help the children to appreciate the inter-relatedness of concepts across the mathematics curriculum, and to view mathematical ideas and concepts as a connected whole. This will enable our pupils to look beyond the basic acquisition of mathematical concepts and skills and inspire them to seek to acquire *"the underlying mathematical thinking and general strategies of problem solving ..."* (Mathematics Syllabus Primary, 2001; p. 5)

Meaningful learning will also help our students to develop a positive disposition toward mathematics. To explore the thematic approach further, we need to examine two very important ideas: abstraction and concepts.

Abstraction and Concepts

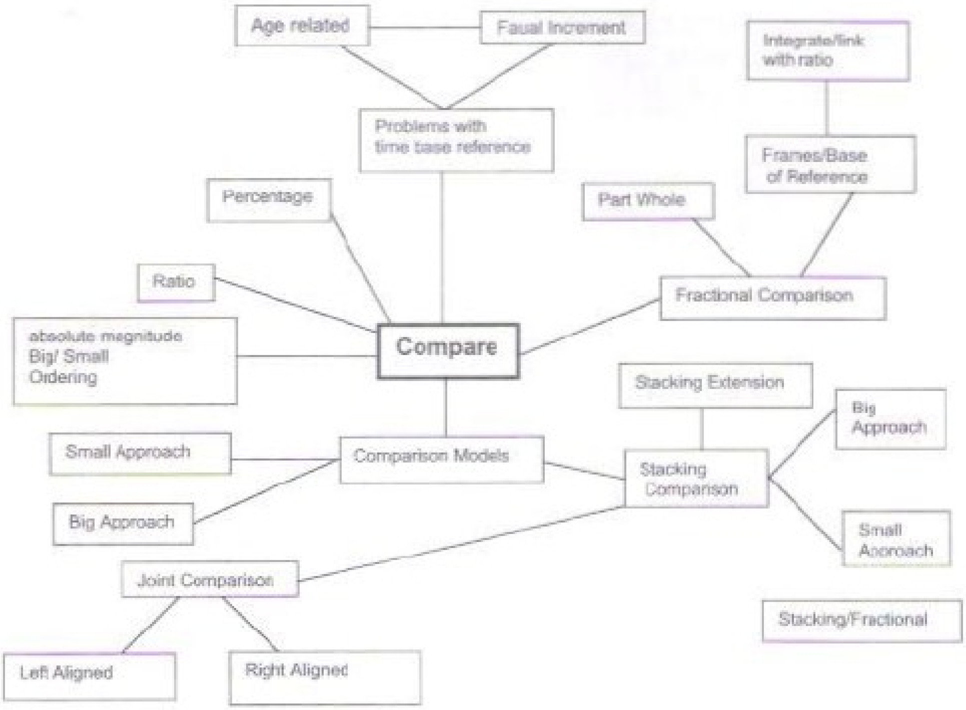
Skemp (1971), in his classic work, *"The Psychology of Learning Mathematics"*, highlighted that from our experiences and repeated encounters of events or problems of the same kind, we abstract invariant properties which persist in memory longer than the specific encounter or experience. An abstraction is therefore some kind of lasting mental image. Abstraction enables us to relate new experiences to a class of related experiences with similar features or characteristics. It enables us to classify. To distinguish abstracting as an activity and abstraction as its end product, Skemp refers to the later as a concept. *"A concept therefore requires for its formation, a number of experiences which have something in common. Once the concept is formed, we may ... talk about examples of the concept."* (Skemp 1971, p. 21)

According to Howard, (1987), ***"A concept is a mental representation of a category, which allows a person to sort stimuli into instances and non instances." (p. 1)***. How one looks at the world depends on one's concepts, these are of fundamental importance in teaching and learning. Concepts are useful as they help us to reduce the complexities of arrays of information into manageable portions. They also enable us to make inferences which may provide insights for the effective solution of problems.

Hence, a great deal of mathematical instruction should involve the teaching of major concepts, how they relate to each other and how to use them. According to Howard (1987), ***"Education is largely a process of teaching students conceptual frameworks that they can use to better understand and deal with the world, and also to learn more about it." (p. 8).*** The use of themes in learning can help to establish such a conceptual framework.

Establishing a Thematic Framework In Mathematics Instruction

A thematic framework can be established by adopting a central theme that allows for inclusion of progressively more challenging and complex concepts. The central theme or set of themes, if well developed, will help students to perceive the relatedness of concepts across a broad spectrum of information or topics. Such perception enables the student to develop a holistic perspective in mathematics learning. The diagram below shows the concept map (see Novak, J.D. 1998) on the theme of **compare.**



Thematic Map for **Compare**

Associating a Strategy / Concept / Ideas with a Name

The above conceptual map will not make very much sense to most of you as the terms used in the map are mathematical vocabulary which are used specifically in my own classrooms. Currently, our pupils are often taught problem-solving strategies without being told what approaches have been used. To facilitate effective communication process in the my classroom, I often associate problem-solving approaches with a suitable term, which may be introduced by me to the pupils, or coined by the pupils themselves. For instance, in my class, a "Japanese Flag" approach evolved, when a child started to use dots to indicate common unitary frame. We also have "Aaron Strategy", which was invented by Aaron. When we talk about "Big Approach" or "Small Approach", everyone knows what they mean. Essentially, what we want is to establish a common vocabulary in mathematics which will help us to facilitate mathematics discourse with greater ease and in greater depth.

Such conceptual linkages when established will:

1. reduce working memory or processing load in problem-solving.
2. facilitate the learning of more advanced concepts which build upon a conceptual notion acquired earlier.
3. facilitate the focus of attention on essential features in a problem.
4. facilitate more effective and meaningful interaction in the classroom.
5. construct linkages and bridges across topics, which help in the integration of core ideas and concepts and identification of recurring themes.

6. illuminate overlapping concepts and overarching principles.

7. facilitate generalisation of concepts and ideas which will lead to

   a) positive transfer of learning
   b) integrative approaches in problem solving
   c) development of greater efficiency in the processing of mathematical information
   d) facilitate a **Whole Part Approach** in problem solving.

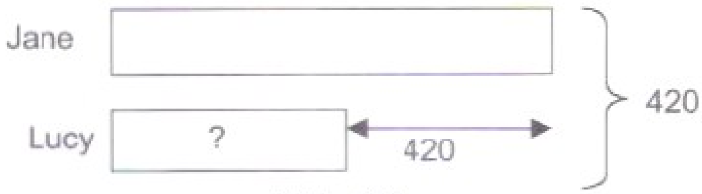
By **Whole Part**, I mean the ability to conceptualise the big picture of a problem before disintegrating the information into its constituent parts. The use of **Part Whole Approach** in problem-solving is very common in the primary school. Relatively fewer pupils are initiated to the **Whole Part Approach**. I have not done any research in this area as yet. However, I am convinced that a child who is trained to conceptualise in terms of the big picture in problem-solving, will develop a more refined understanding of mathematics in the long term and will become a more efficient thinker.

Let me illustrate the significance of the theme: **compare** (seen in the thematic map given earlier) and the idea of associating a strategy with a conceptual notion with the following examples:

Example 1

Comparison Model

Small Approach (because the unknown is the small rectangle)
Jane and Lucy have a total 800 stamps. Jane has 420 more stamps than Lucy. How many stamps does Lucy have ?



$$\frac{800 - 420}{2}$$

$$= \frac{380}{2}$$

$$= 190$$

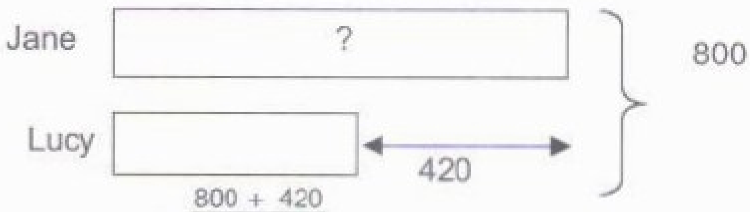
Lucy has 190 stamps.

Example 2

Comparison Model

Big Approach

Jane and Lucy have a total 800 stamps. Jane has 420 more stamps than Lucy. How many stamps does Jane have?



$$\frac{800 + 420}{2}$$

$$= \frac{1220}{2}$$

$$= 610$$

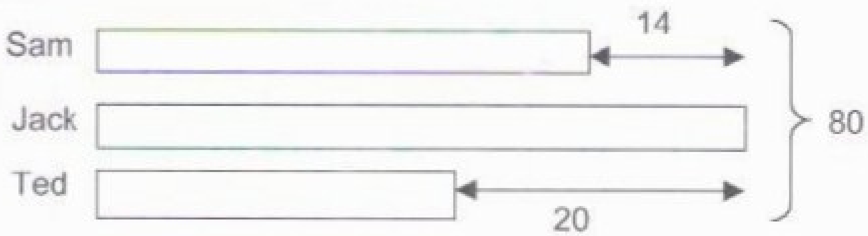
Jane has 610 stamps.

The concept of Comparison Model can also be extended to following problem which can be solved using the idea of Big Approach and finding the complement.

Example 3

Comparison Models

Complementary Approach (Extension to Big Approach)
Jack, Sam and Ted have a total of 80 stamps. Jack has 14 more stamps than Sam and 20 more stamps than Ted. How many stamps do Sam and Ted have altogether?



$$\frac{80 + 14 + 20}{3}$$

$$= \frac{114}{3}$$

$$=38$$

Jack has 38 stamps.

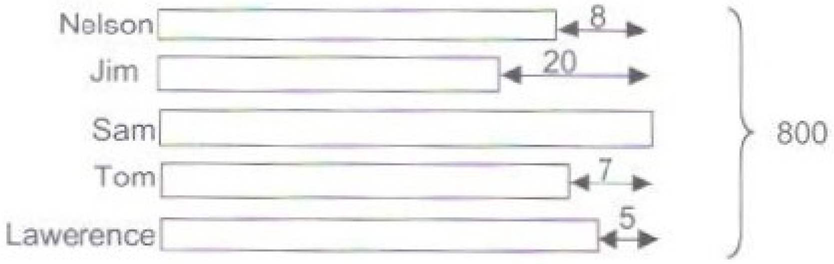
80 — 38 = 42

Sam and Ted have a total of 42 stamps.

Example 3 is a natural extension of Example 2, incorporating the concept of complement. We intend to find the total number of marbles of Sam and Ted, however, it is easier to focus on the complement, which is Jack's marbles, by utilising the concept of Big Approach. This question can be given as a quiz, which will provide a new challenge to the children. Yet, it is within the grasp of children with a firm conceptual understanding of the Big Approach. This is the most important feature of the thematic framework. There is a need to select problems which build upon the existing knowledge of students. Such problems should ideally allow for extension of the student's knowledge, and yet are manageable because they build upon the previous concepts established. How this should be done, depends on the particular cohort of students. A theme should be developed, extended and expanded according to the competence level of the students, in order to cater to their specific learning needs. Let us now look at Example 4.

Example 4

Sam has 20 more marbles than Jim and 7 more marbles than Tom. Nelson has 8 fewer marbles than Sam, while Lawrence has 5 fewer marbles than Sam. The children have a total of 800 marbles. Find the average number of marbles that Jim, Tom, Nelson and Lawrence have.



$$\frac{800 + 8 + 20 + 7 + 5}{5}$$

$$= \frac{840}{5}$$

$$= 168$$

Sam has 168 marbles.

$$\frac{800 - 168}{4}$$

$$= \frac{632}{4}$$

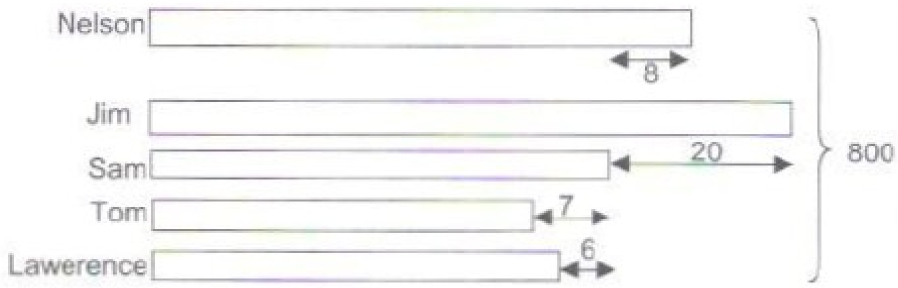
$$= 158$$

The average number of marbles of Nelson, Jim, Tom and Lawrence is 158.

Example 4 looks frighteningly difficult. However, it is a mere extension of the Complement Approach, with a further extension by incorporating the concept of average. This question can also be done with the Small Approach. However, it will be very tedious. There is therefore a great advantage to introduce alternative perspectives of problem-solving to the students. Let us now look at one last illustration.

Example 5

Jim has 20 more marbles than Sam, while Nelson has 8 more marbles than Sam. Sam has 7 more marbles than Tom and 6 more marbles than Lawrence. The children have a total of 800 marbles. Find the average number of marbles of Jim, Sam, Nelson and Lawrence. Correct your answer to the nearest whole number.



$$\frac{800 - 8 - 20 + 7 + 6}{5}$$

$$= \frac{785}{5}$$

$$= 157$$

Sam has 157 marbles.

$$157 - 7 = 150$$

Tom has 150 marbles.

$$\frac{800 - 150}{4}$$

$$= \frac{650}{4}$$

$$= 162.5$$

$$\approx 163 \text{ to 1 dec place)}$$

Nelson, Jim, Sam and Lawrence each has an average of 163 marbles.

A cursory glance will cause some students to claim that Example 4 and 5 are the same. However, their difference is rather great. In Example 4, a combination of Big Approach and Small Approach is required to find Sam's marbles, after which Tom's marbles (the complement) need to be computed before the average of the other four boys can be found. Another extension here is the introduction of 'rounding off' into the question.

Space does not permit me to elaborate on all the other extensions in the above thematic framework. However by now, the general idea of how to develop such a framework is clear. I will leave it to the reader to explore other possibilities for extension. I will now move on to the next important idea that can facilitate a natural expansion of any thematic framework: Integrative Perspective in Problem-Solving.

Integrative Perspective in Problem-Solving

Our current mathematics text is very compartmentalised. Mathematics is demarcated into neat topics: whole numbers, fractions, decimals, ratio and proportion, percentages, ... etc. This is necessary in the initial stages of learning. However, very little is done to present an integrative view of problem-solving to the children in the long run. This is a major weakness that should be addressed. As the children mature in their ability to conceptualise mathematical ideas, we should provide opportunities for them to wander across the frontiers of topical boundaries and allow them to explore the varied possibilities in handling a problem using concepts from across different topics. There are many problems that can be solved by various approaches using concepts from different topics. For instance, some percentage problems can often be transformed to a unitary correspondence problem with the ingenious use of models. Other percentage problems can be solved efficiently by using ratio and proportion. We need to provide adequate activities at the upper primary (primary five and primary six) to help the children become aware of the connections and linkages across topics. Then they will develop a broad perspective of mathematics which will facilitate a helicopter view in problem-solving.

Communication in Mathematics

I highlighted communication in the Pentagonal Framework because mathematical communication plays a very important role in the development of any thematic framework. At this time, across all topics, mathematics communication is often seen as a side issue. Children are often not taught how to use mathematical expressions effectively to communicate their ideas. Hence, there is a tendency for them to use a series of fragmented expressions in problem-solving, often without the support of any statement to convey their thinking processes. This is especially evident in geometry, where children are taught how to find the unknown angles in various geometry situations, yet they are not required to explicitly justify their steps with qualification. (Reason given being that they are too young to present their answers with such rigour). Pedagogically, this is unsound. Such practice leads to procedural competence without a corresponding development in conceptual understanding. Children who are able to derive the answer to a geometrical question but are not able to put forth their solution convincingly with rigorous justification are not effective in using mathematics as a means for communication. Such children will often be unable to extend their

web of conceptual knowledge and relate them in a flexible, meaningful ways to new information and situations. We often observe such inflexibility in pupils who are thrown off by the slightest twist in the structure or formulation of a problem.

My conviction is when there is a need for justification and clear communication, and when this need is set as an expectation for a reasonable degree of rigour in presentation of mathematical solution with appropriate communication, learning will be greatly enhanced in the long term. We need to have faith in our children. They are capable of communicating clearly in mathematics. Moreover, sloppy habits once cultivated are hard to change. It is difficult to teach children who are not used to a reasonable rigour in presentation of mathematics to present their work properly in the secondary schools. Hence, we must not be myopic in our approach. Proper structure and presentation in mathematics should be taught from young, so that our children will develop the capability to communicate mathematics clearly in the long run. Teaching should be seen as a joint enterprise among primary schools, secondary schools, junior colleges and universities. In this sense, we must not merely prepare our children to do well in the PSLE, but also prepare them adequately so that there will be positive transfer of learning from the primary to the secondary schools.

The unfolding of a mathematics lesson is never totally predictable. No lesson, however well planned can unfailingly lead the class smoothly through different objectives of learning at all times. There are times when we need to be flexible and modify our approach, or even abandon pre-planned activities when feedback from our pupils tell us that our teaching activities are not appropriate. There are other times, when instead of rushing on to the next activity in our agenda, we should grasp hold of incidental spin offs from a lesson, which provides opportunities for in-depth discussion of proposed solutions, analysis of methods of attacking problems and formulation of over-arching concepts. Such precious teaching moments can often be creatively harnessed to bring about extensive collateral learning which is of greater value than the pre-planned activities. It is also essential to encourage our pupils to share their ideas through presentations on the board and peer discussion; and provide avenues for them to probe, analyse, evaluate and refine each other's problem solving approaches. Creative solutions or insights of children can also be refined and incorporated into a thematic framework. In this sense then, an ideal thematic framework is a dynamic one, which allows for refinement as the teaching and learning process unfolds.

Personally, I believe that communication is a life-skill which should be incorporated into every subject. This is reflected in the words of Skemp, (1971). *"Communication seems to emerge as one of the influences favourable to the development of reflective intelligence. One of the factors involved is certainly the necessity to link ideas to symbols: .... Another is the interaction of one's own ideas with those of other people. To the extent that agreement is reached, the resulting communality of ideas is less egocentric, more independent of individual experience. .....* ***the cut and thrust of intellectual discussion forces one to clarify one's own mind, to state them in***

terms not likely to be misunderstood, to justify them by revealing their relationships with other ideas; and also to modify them when weaknesses are found..." *[p. 61 – 62]*

Toward Autonomy

I will conclude with some thoughts on autonomy. Outstanding teachers over the ages, judge their own success by having equipped their students to stand on their own. *"The final contribution of the excellent teacher is however, gradually to reduce the learner's dependence...."* (Skemp, 1971, p. 73). Wilkerson (1992) puts it succinctly in another way, *"The ultimate goal of equipping is independent equippers."* Is such a goal too far fetched and ambitious, some may ask. Personally, I believe that a teacher must teach with a conviction that he is able to impact the very way the children think by orchestrating the learning situation for conducive learning to take place. The best teachers are those who are ever ready to relinquish their hold on their students, whenever the students are ready to stand on their own. It is not a matter of holding the hand all the way and then letting go totally. It is more of holding and letting go at appropriate intervals. When children need support, motivation and encouragement, our helping hands must be there. In instances where the challenge is within the ability of our students' capabilities, we must recede into the background. How autonomy can be fostered and developed or encouraged effectively is an area which needs further research and exploration, but as a member of the joint enterprise of Education, teachers at all levels can each contribute their part.

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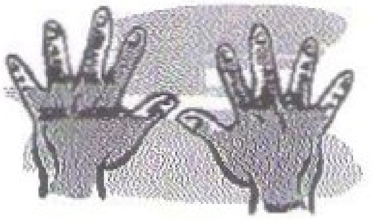
Teachers' Clipboard...

"Teaching kids to count is fine, but teaching them what counts is best."

- Bob Talbert -



"Imagination is more important than knowledge."

- Albert Einstein -

"In school we add, multiply and subtract.
Sometimes teachers don't like the way we act. But if we are nice to them,
they'll be nice to you.
For it all depends
on who is nice to who."

- Anonymous student

age 8 -

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